Signals and its properties

Signals and codes (SK)

Department of Transport Telematics
Faculty of Transportation Sciences, CTU in Prague

Lecture 1

Lecture goal and content

Goal

- Understand what is signal, know basic types of signal and their basic characteristic values and reveal that signals are everywhere around us.

Content

- Signals
  - what is it?
  - types of signals
  - examples of signals
  - characteristic values of signals
    - instantaneous value
    - average value
    - signal energy
    - signal power
    - effective value

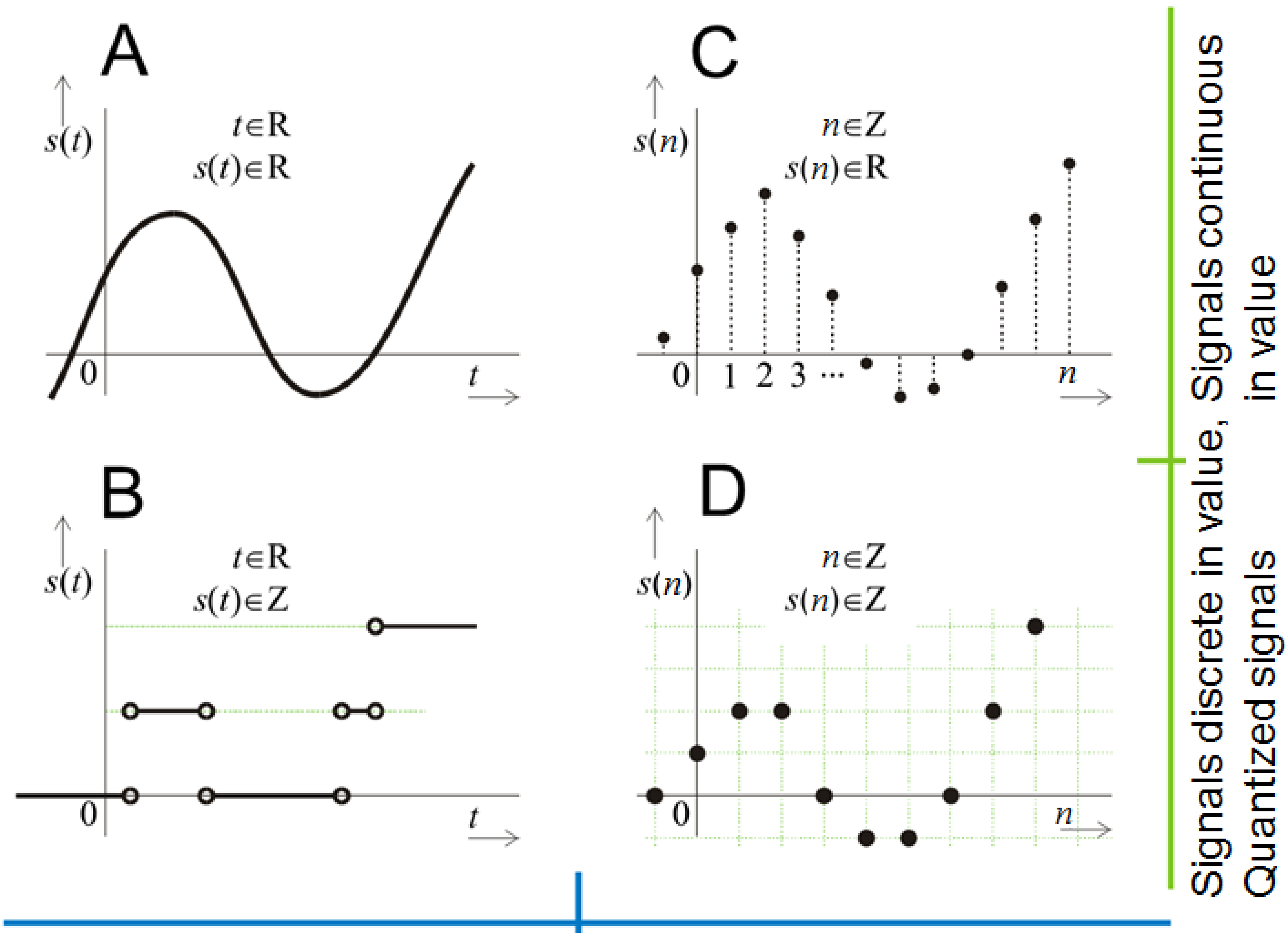
What is signal?

Definition: an abstraction of any measurable quantity that is a function of one or more independent variables such as time or space. For this course it is some function of time.

Examples:

- A voltage or a current in a circuit
- Electrocardiograms
- Sinusoid $A.\sin(\omega t + \varphi)$
- Speech/music
- Intensity of light radiation
- Acoustic pressure
- Image, video
- etc.

Signal types: continuous (C), discrete (D)



Signals in continuous time

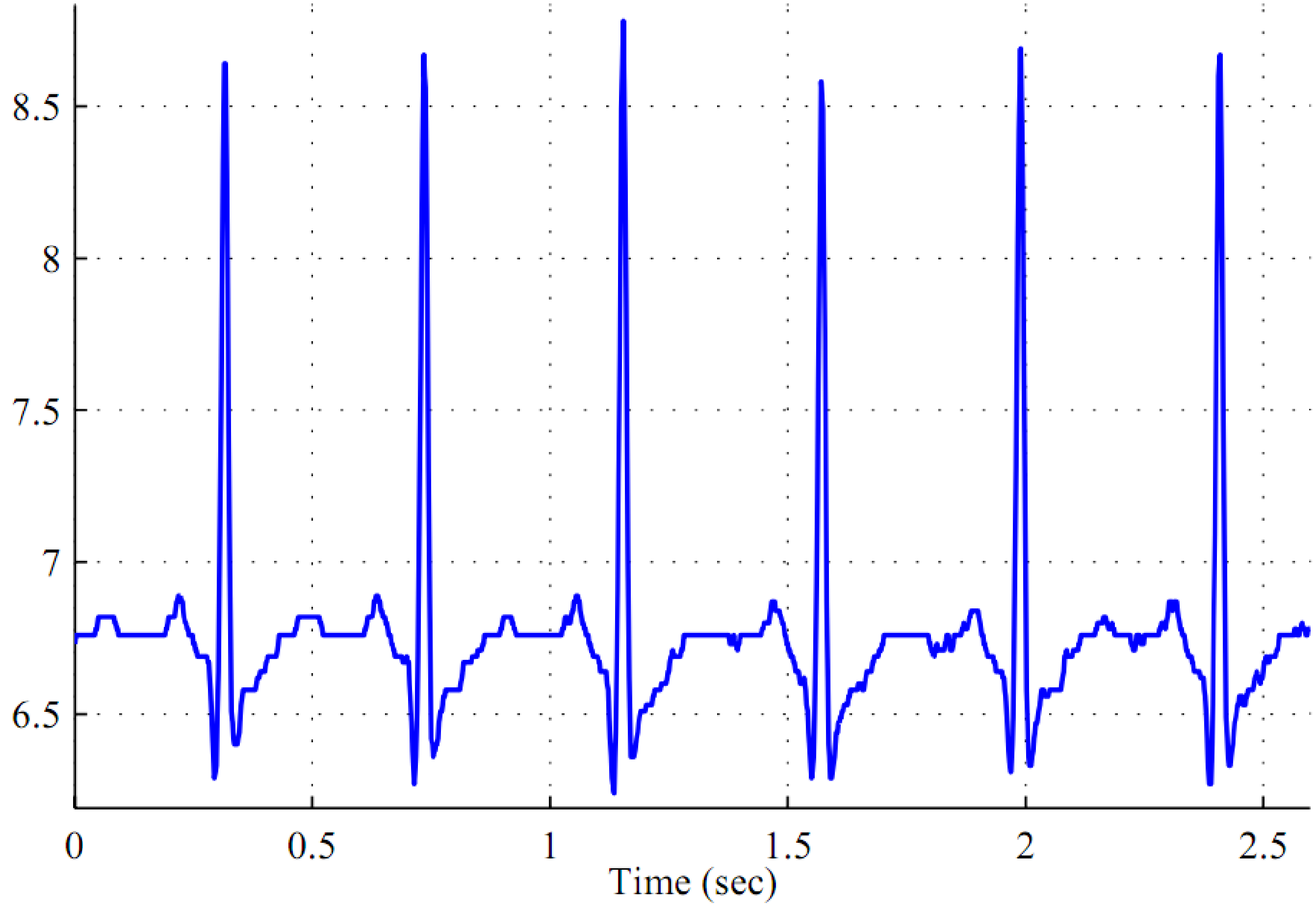
Signals in discrete time, Sampled signals, sequences

Basic signal operations

- Time shift: $s(t-t_0)$ and $s[n-n_0]$
  - If $t_0 > 0$ or $n_0 > 0$, signal is shifted to the right
  - If $t_0 < 0$ or $n_0 < 0$, signal is shifted to the left
- Time reversal: $s(-t)$ and $s[-n]$
- Time scaling: $s(at)$ and $s[an]$
  - If $a > 1$, signal is compressed
  - If $1 > a > 0$, signal is stretched
- Signal scaling: $as(t)$ and $as[n]$
  - If $a > 1$, signal has higher values
  - If $1 > a > 0$, signal has lower values

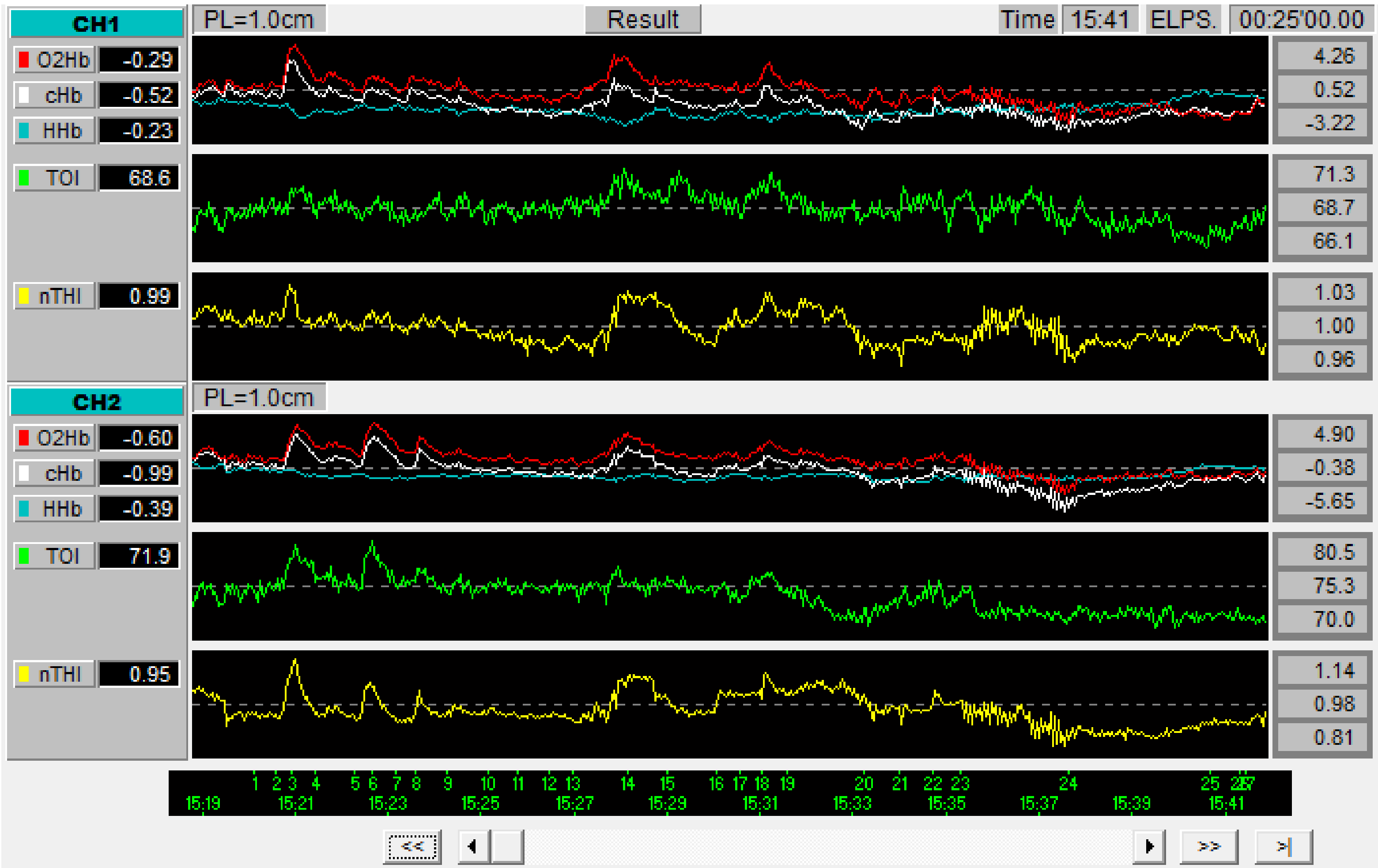
Signal examples

- ECG (electrocardiogram), in Czech EKG



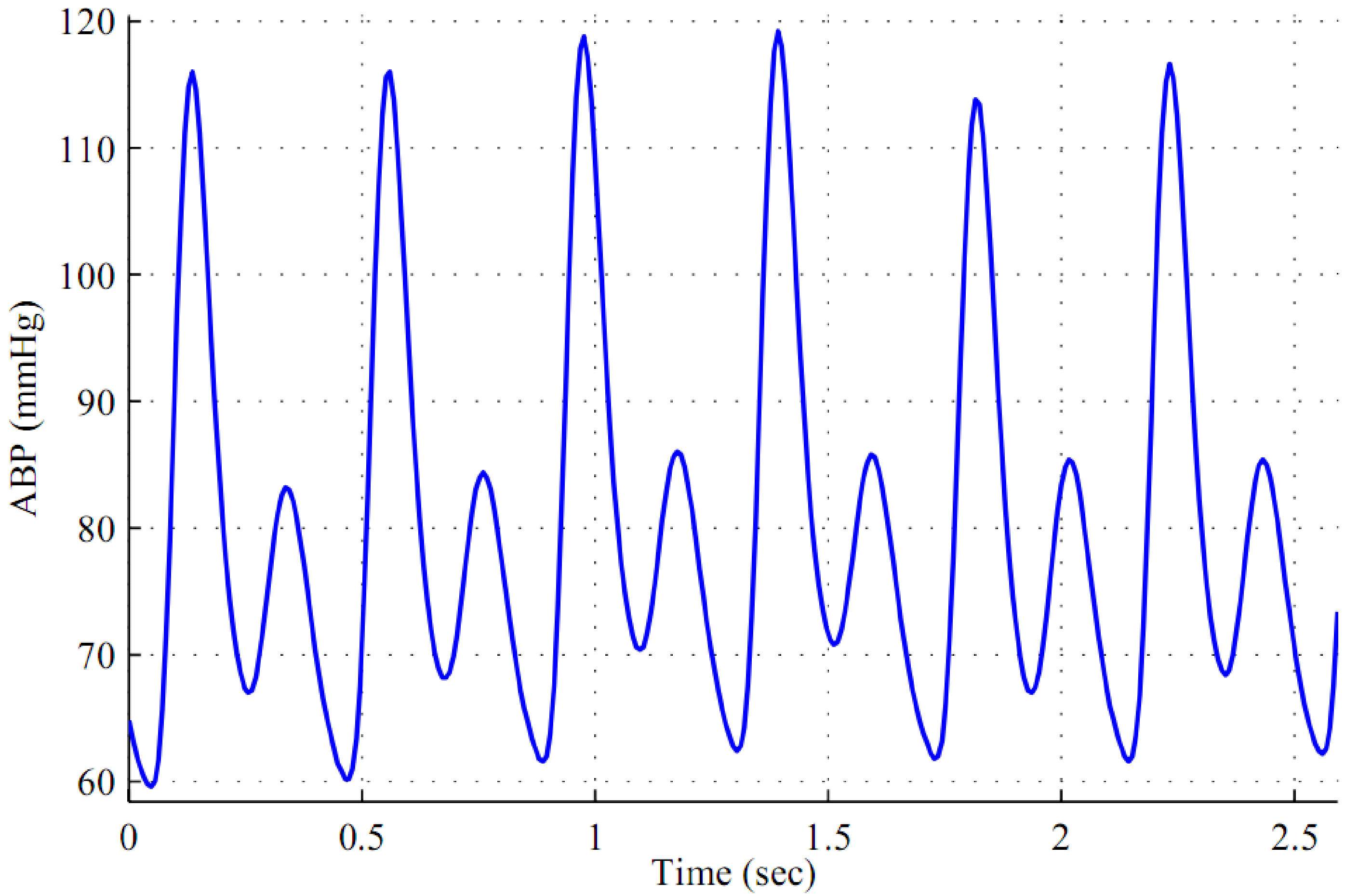
Signal examples

- EEG (electroencephalogram)



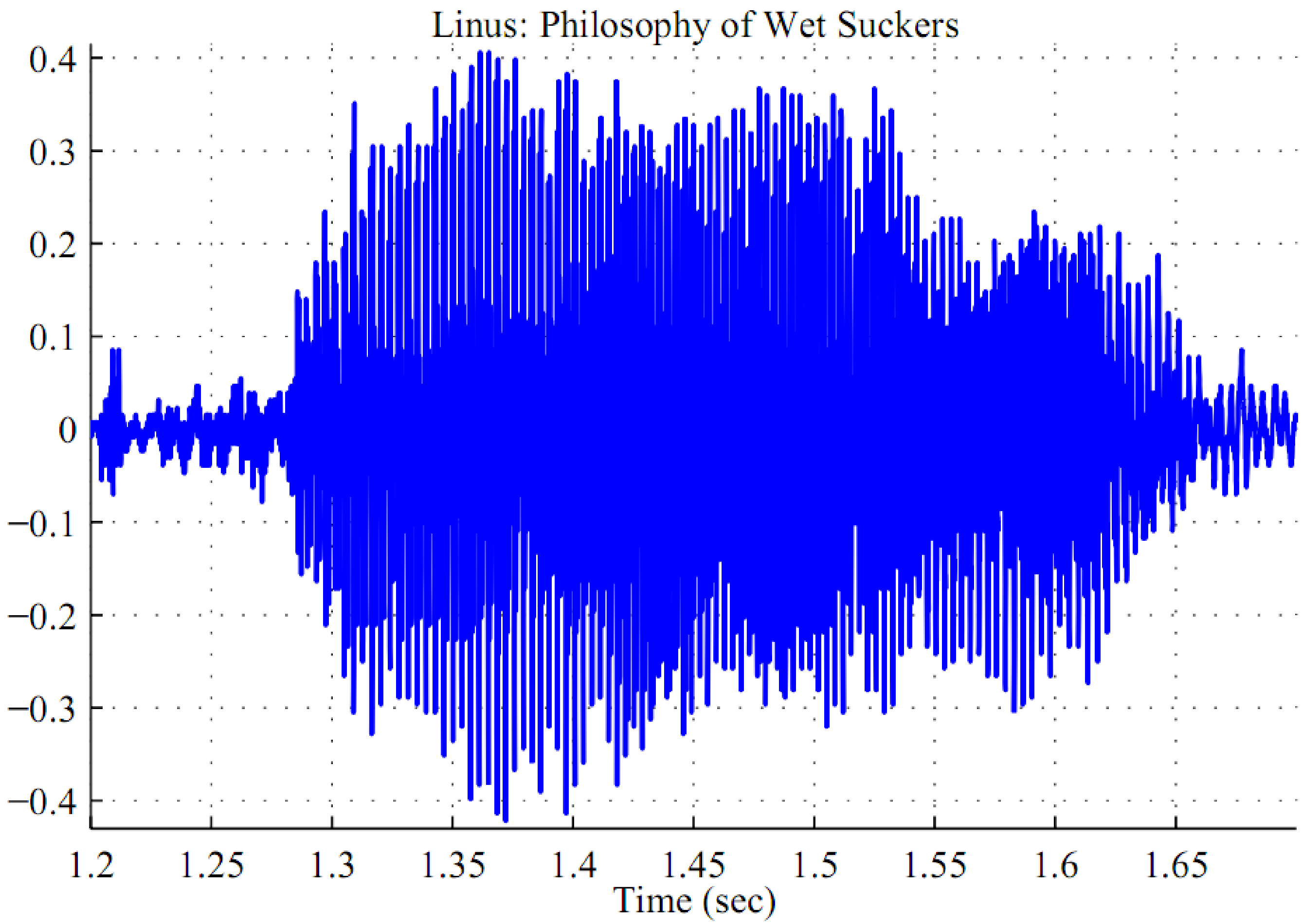
Signal examples

- Arterial pressure (tepenný tlak)



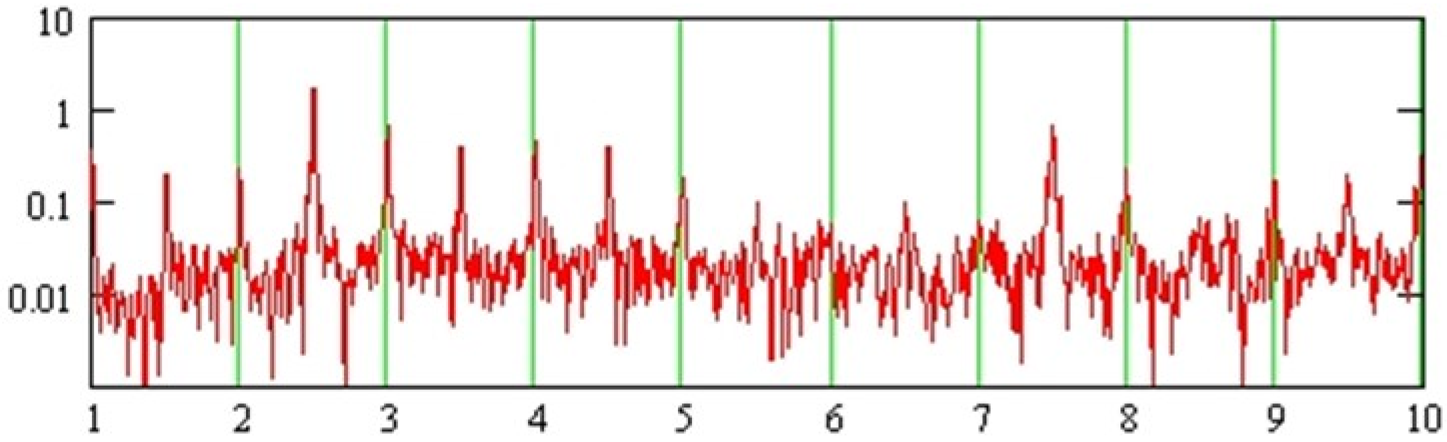
Signal examples

- Speech (řeč)



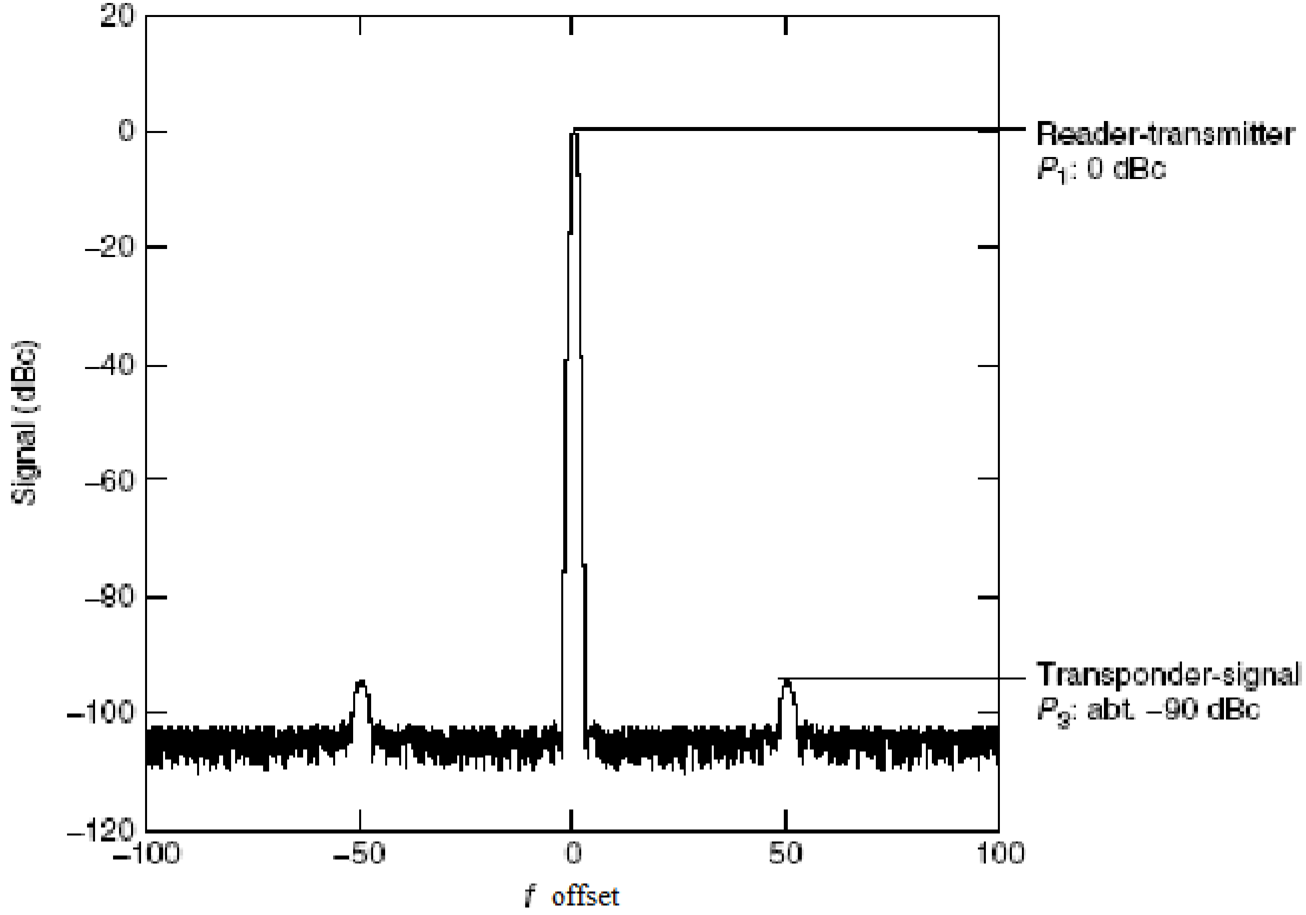
Signal examples

- NVH (Noise, vibration, and harshness) (hluk a vibrace)
  - Here is the noise **spectrum** of Michael Schumacher's Ferrari at 16680 rpm, showing the various harmonics. The x axis is given in terms of multiples of engine speed. The y axis is logarithmic, and uncalibrated (https://en.wikipedia.org/wiki/Noise,_vibration,_and_harshness).



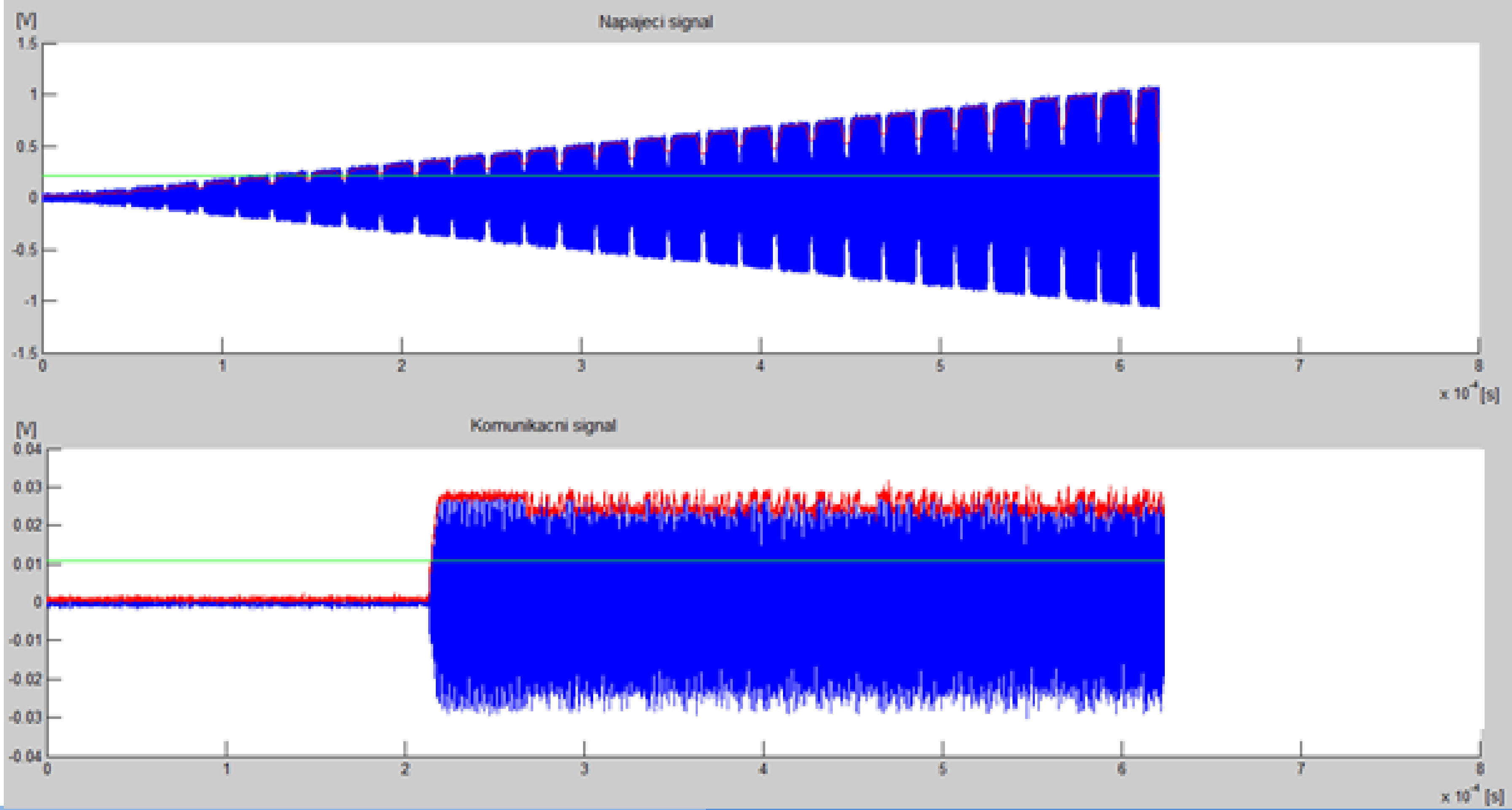
Signal examples

- RFID (not only in transport)



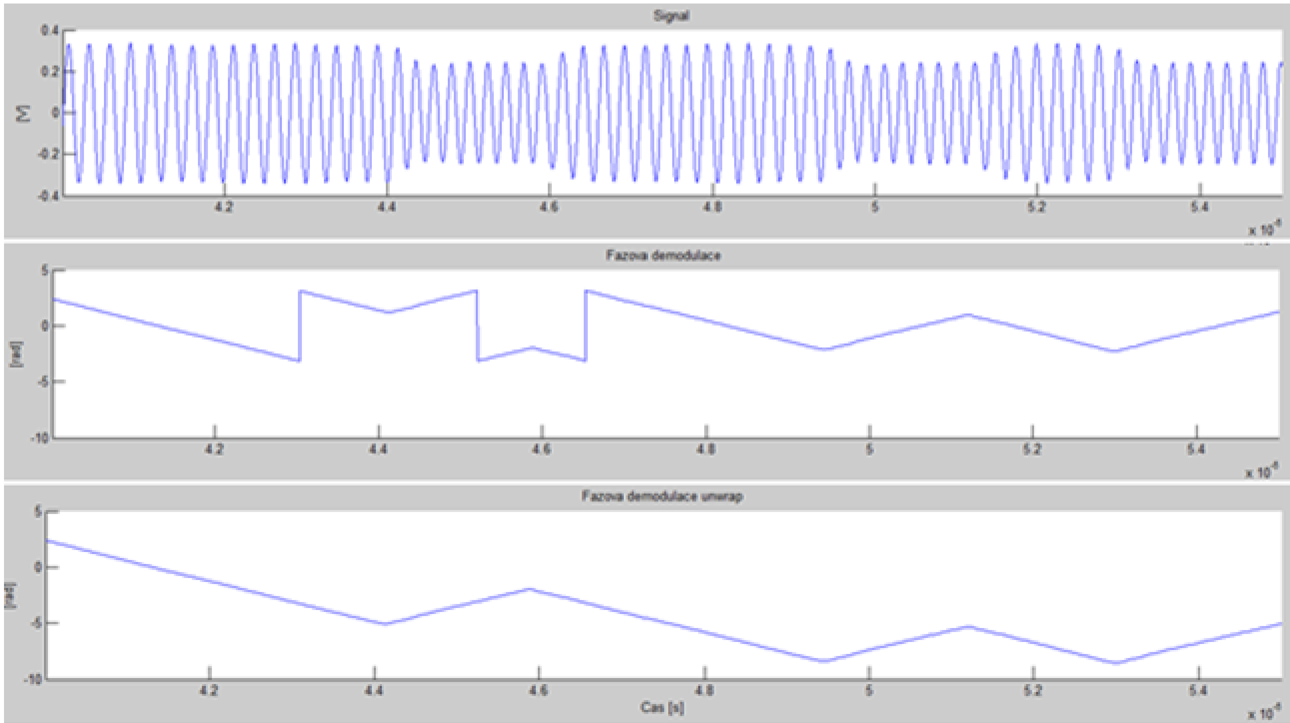
Signal examples

- Eurobalise – AM modulated tele-powering signal and up-link signal (balise responce) (napájecí a komunikační signál (odpověď balízy))

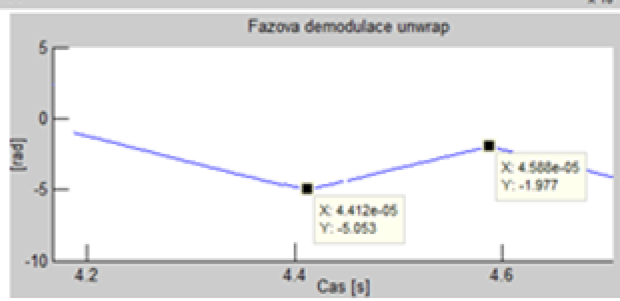


Signal examples

- Eurobalise – phase demodulation of FSK modulated up-link signal

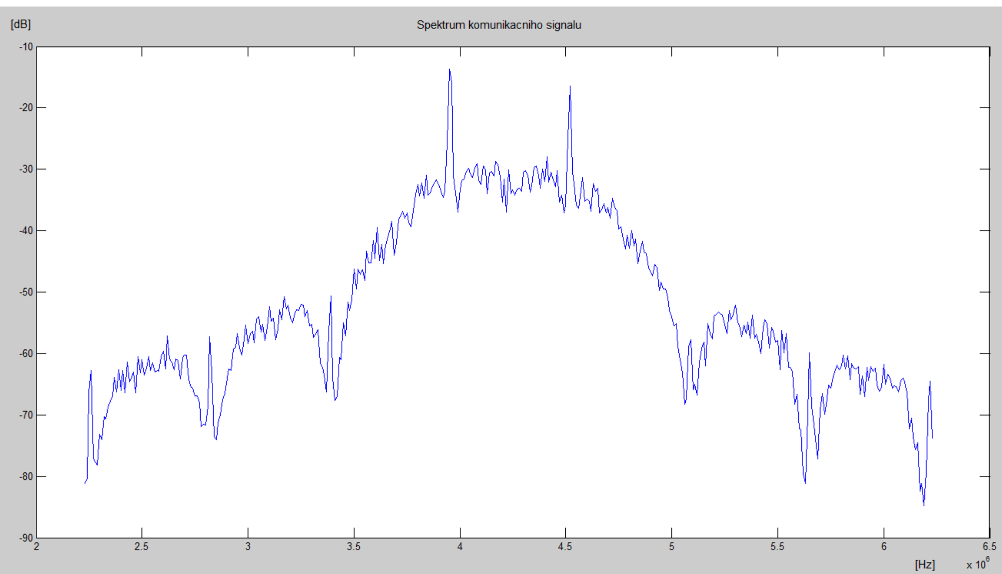


$$f = \frac{\omega_1 - \omega_2}{2.\pi.(t_1 - t_2)} + f_{\text{centre}}$$

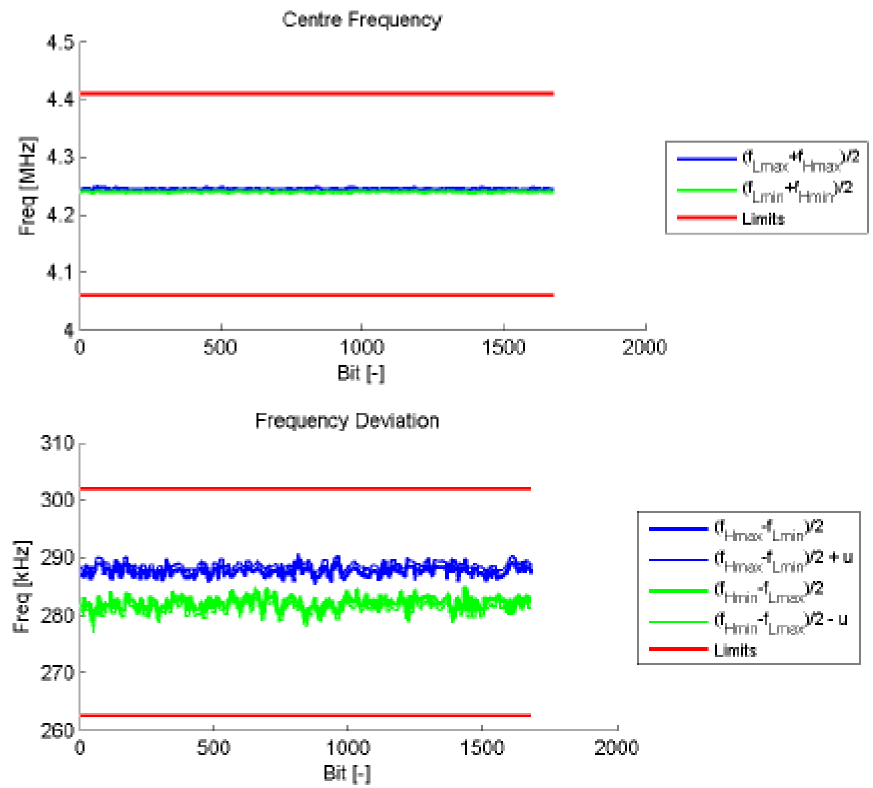


Signal examples

- Eurobalise – spectrum of up-link signal

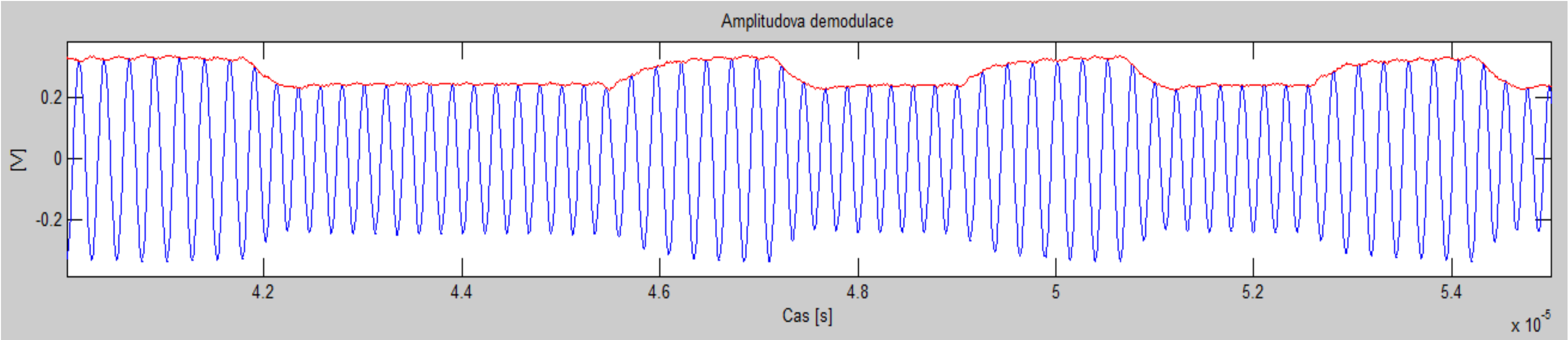


- Eurobalise – up-link signal: centre frequency and deviation



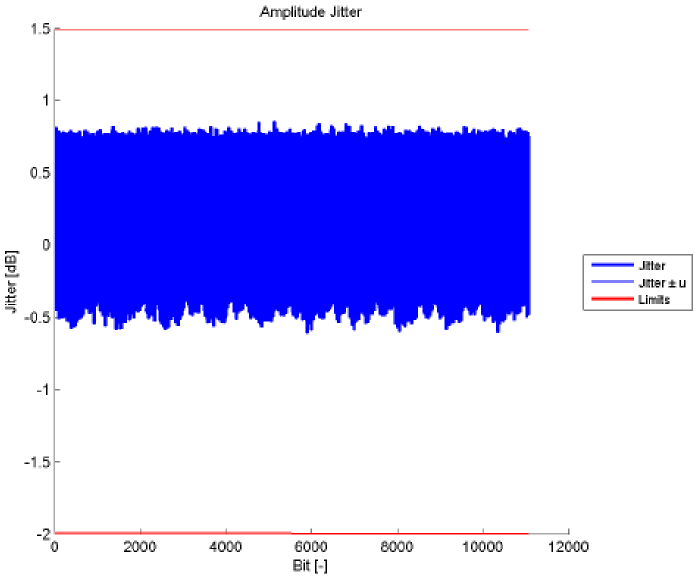
Signal examples

- Eurobalise – amplitude demodulation of up-link signal



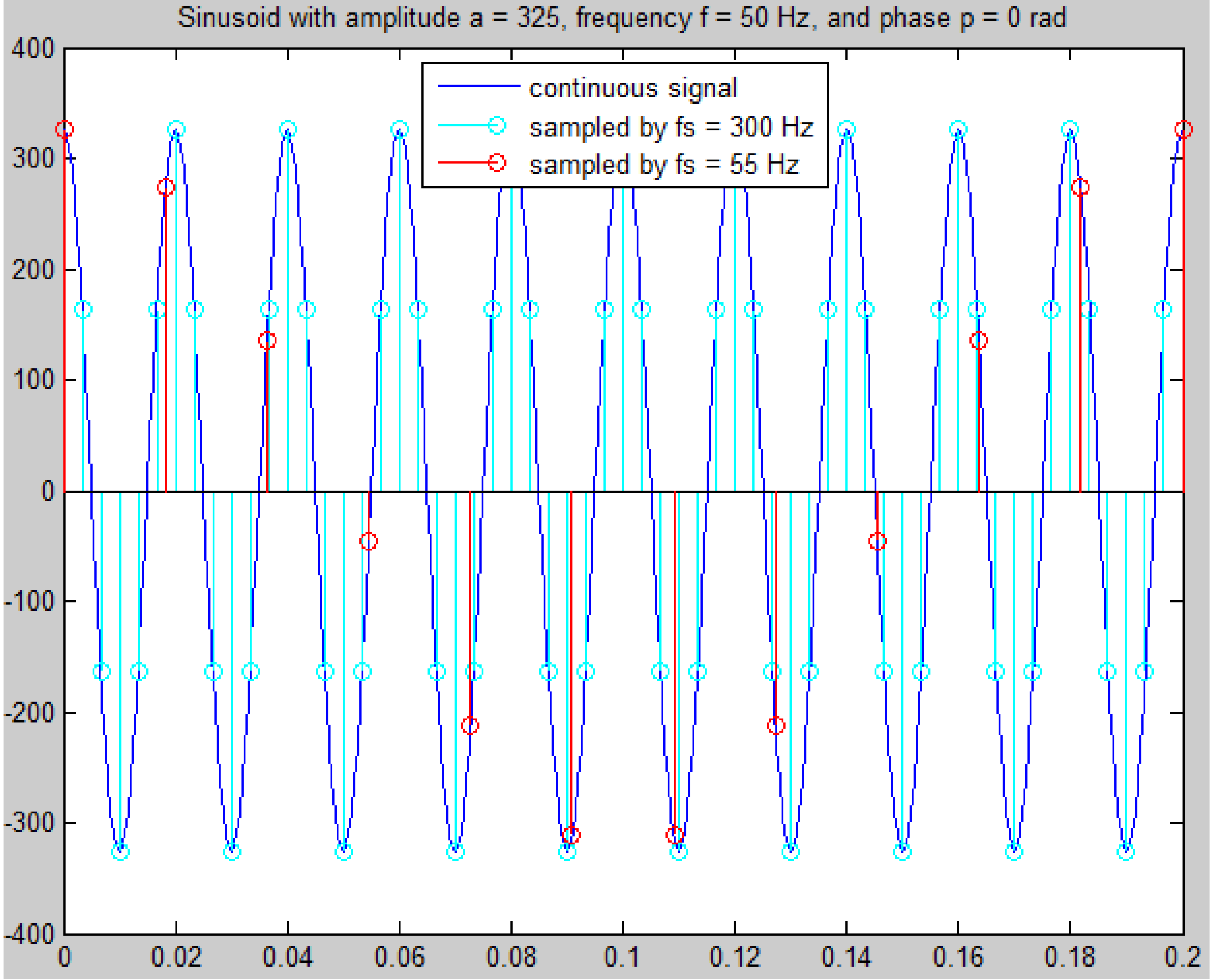
- Eurobalise – amplitude jitter

(kolísání amplitudy)



Signal examples

- Sampled socket voltage 230 V



Signal examples

- Video detection of vehicles

pictures from https://www.youtube.com/watch?v=F_M_skebbpA

Characteristic values of signals $x(t)$

1. Instantaneous value $x(t_i)$, $x[n_i]$

Exe.1.1: Given signal $x(t) = 325 \cdot \sin(2\pi \cdot 50 t)$

Find instantaneous value of the signal for time instant $t = 10$ ms.

Sol.: $x(10 \cdot 10^{-3}) = 325 \cdot \sin(2\pi \cdot 50 \cdot 0{,}01) = 325 \cdot \sin\pi = \underline{0}$ ∎

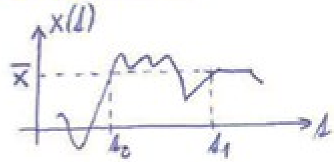
Note: x is dimensionless

Q: Is socket voltage 230 V safe at time instant $t = 10$ ms?

A: Yes, BUT dangerous touch takes $t > 0$ s, see RMS value below.

2. Average value $\bar{x}$ in a time interval

- Continuous time (CT):

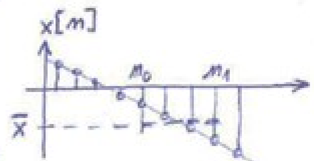


$$\bar{x} = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} x(t)\, dt$$

- periodic function: choose $t_1 - t_0 = T_0$ ... fundamental period

arbitrary time instant thus $\bar{x} = \frac{1}{T_0} \int_{t_i}^{t_i + T_0} x(t)\, dt$

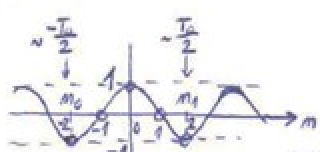
- Discrete time (DT)



$$\bar{x} = \frac{1}{n_1 - n_0 + 1} \sum_{n=n_0}^{n_1} x[n]$$

- periodic function: choose $n_1 = n_0 + \frac{T_0}{T_S} - 1$ (signal period / sample period)

thus: $\bar{x} = \frac{T_S}{T_0} \sum_{n_0+1}^{n_0 + \frac{T_0}{T_S}} x[n]$



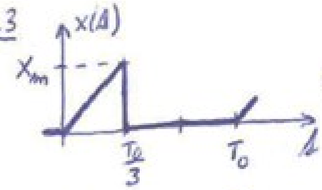
[01-1]

Exe.1.2: $x(t) = e^{-0{,}2t}$ Find average value $\bar{x}$ in time interval $t \in \langle 0; 2 \rangle$ s.

(Note: Transient phenomena equation ... $e^{-\frac{t}{\tau}}$, thus $\tau = 5$ s)

Sol.: $\bar{x} = \frac{1}{2-0} \int_0^2 e^{-0{,}2t}\, dt = \frac{1}{2}\left[\frac{-1}{0{,}2} \cdot e^{-0{,}2t}\right]_0^2 = \frac{-5}{2}(e^{-0{,}4} - 1) \doteq \underline{0{,}8242}$ ∎

Exe.1.3



Find average value of the signal $x(t)$ (note: time interval = one fundamental period T_0) (note: result is apparent at first glance)

Sol.: $\bar{x} = \frac{1}{T_0} \int_0^{T_0} x(t)\, dt = \frac{1}{T_0} \int_0^{\frac{T_0}{3}} \frac{X_m}{\frac{T_0}{3}} \cdot t\, dt = \frac{3X_m}{T_0^2}\left[\frac{t^2}{2}\right]_0^{\frac{T_0}{3}} = \frac{3X_m}{T_0^2} \cdot \frac{T_0^2}{3^2 \cdot 2} = \underline{\frac{X_m}{6}}$

Exe.1.4: $x[n] = 325 \cdot \sin\left(2\pi \cdot 50 \cdot \frac{1}{200} \cdot n\right)$ $\frac{1}{f_S} = T_S$

Find average value of given discrete time periodic signal.

Sol.: $T_0 = \frac{1}{f_0} = \frac{1}{50}$ s, $T_S = \frac{1}{200}$ s $\Rightarrow \frac{T_0}{T_S} = 4$, $n_0 = 0$, $n_1 = 3$

$$\bar{x} = \frac{1}{3-0+1} \sum_{n=0}^{3} 325 \cdot \sin\left(\frac{\pi}{2} \cdot n\right) = \underline{0}$$ ∎

3. Signal energy E

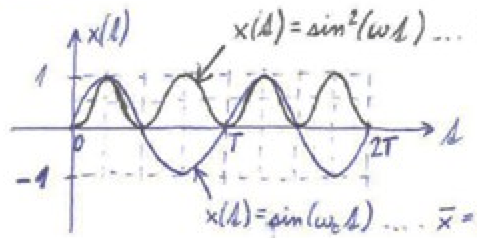
← energy signals have $0 < E < \infty$

CT: $E = \int_{-\infty}^{\infty} |x(t)|^2\, dt$

DT: $E = \sum_{n=-\infty}^{\infty} |x[n]|^2$

note: absolute value in the formulas is important for complex signals (otherwise is not necessary)

Demo:



$x(t) = \sin^2(\omega t)$ ... $\bar{x} = \frac{1}{2}$, $E = \infty$ (adding values forever, E is not proper characteristic value)

$x(t) = \sin(\omega_0 t)$ ... $\bar{x} = 0$, but there is some energy

4. Signal power P ← power signals have $0<P<\infty$

- (average) value of signal energy over a time interval (usually period). (note: physics analogy: $p=\frac{dW}{dt}$)

General CT signals:

$$P=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}|x(t)|^2\,dt$$

General DT signals:

$$P=\lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{+N}|x[n]|^2$$

Periodic CT signals:

$$P=\frac{1}{T_0}\int_{t_i}^{t_i+T_0}|x(t)|^2\,dt$$

Periodic DT signals:

$$P=\frac{1}{N}\sum_{n=n_i+1}^{n_i+N}|x[n]|^2, \text{ where}$$

$N=\frac{T_0}{T_S}$ … samples per period

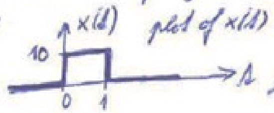
5. Effective value of a signal X_{RMS} (root mean square)

$$X_{RMS}=\sqrt{P}$$

- equivalent constant signal with the same power as $x(t)$

Exe. 1.5: Find energy, power and effective value of signals:

a) $x(t)=\begin{cases}10 & \dots\ 0\le t\le 1\\ 0 & \dots\ \text{otherwise}\end{cases}$ (plot of $x(t)$)

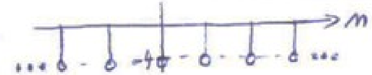


Sol.: $E=\int_{-\infty}^{\infty}|x(t)|^2\,dt=\int_0^1 10^2\,dt=100\cdot[t]_0^1=\underline{100}$

$$P=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}|x(t)|^2\,dt=\lim_{T\to\infty}\frac{1}{2T}\cdot 100=\underline{0}$$

$X_{RMS}=\sqrt{P}=\underline{0}$ ■

b) $x[n]=-4\ \forall n$ (stem plot:



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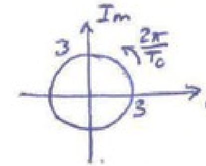
Sol.: $E=\sum_{n=-\infty}^{+\infty}|x[n]|^2=\infty\cdot 16=\underline{\infty}$

$$P=\lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{+N}|x[n]|^2=\lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{+N}|-4|^2=\lim_{N\to\infty}\frac{2N+1}{2N+1}\cdot 16=\underline{16}$$

$X_{RMS}=\sqrt{P}=\underline{4}$ ■

c) $x(t)=3\cdot e^{j\frac{2\pi}{T_0}t}$ (polar plot:



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Sol.: $E=\int_{-\infty}^{\infty}|x(t)|^2\,dt=9\cdot\infty=\underline{\infty}$

$$P=\frac{1}{T_0}\int_0^{T_0}|x(t)|^2\,dt=\frac{1}{T_0}\int_0^{T_0}\left|3\cdot e^{j\frac{2\pi}{T_0}t}\right|^2 dt=\frac{1}{T_0}\cdot 9\cdot[t]_0^{T_0}=\underline{9}$$

$X_{RMS}=\sqrt{P}=\underline{3}$ ■

Energy vs. power – questions and answers

Q: Is some energy signal also power signal?

A: No. If $0<E<\infty$, then $P=0$. If $0<P<\infty$, then $E=\infty$.

Q: Are all signals either energy or power signals?

A: No. Any infinite duration increasing amplitude signal will not be either. (example: $x(t)=t^2$ is neither power signal ($P=\infty$) nor energy signal ($E=\infty$).)

Vocabulary:
- instantaneous value – okamžitá hodnota
- time instant – časový okamžik
- average value – průměrná hodnota
- fundamental period – základní perioda
- arbitrary – libovolný
- transient phenomenon – přechodný děj

01-2

References

- Vejražka, František. Signály a soustavy / 4.vyd. Praha: ČVUT, 1996. 243 s. ISBN 80-01-00450-3., In Czech