

20SK: Exercise #6

Channel Coding (ECC)

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Introduction

Source encoding: *Remove redundant information.*

Channel coding: *Protect data against transmission errors.*

Medium: *Controlled redundancy (increase message entropy).*

Always **block codes**

Linear codes

- ▶ Repetition code
- ▶ Hamming code
- ▶ BCH
- ▶ LDPC (Gallager, 1963)
- ▶ Cyclic codes (Reed-Solomon)

Other approaches:

- ▶ Convolution codes
- ▶ Turbo codes

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 - 2.3 probability that an error occurs in any k positions is a binomic number
 - 2.4 we have to sum for every possible length up to n , hence

$$p(n, \epsilon) = \sum_{k=(n+1)/2}^n \binom{n}{k} \epsilon^k (1 - \epsilon)^{n-k}.$$

Problem 1 continued

3. Create function `rcbsc(n,ep)` computing error probabilities for odd length up to n and plotting them using `stem()` into a graph.
4. Q: What would be the optimal length of the repetition code?
5. Q: What would be the code rate and transmission speed?
6. Q: How does it correspond to Shannon limit theorem? Discuss the difference.

Problem 2

Assume linear code with a generator matrix \mathbf{G} .

1. Q: Given \mathbf{G} , what is the length of plain text vector \mathbf{v} ?
2. Q: How are codewords computed?
3. Q: How will you enumerate all codewords?
4. Write function `[C,dmin]=linprop(G)` that computes a binary matrix \mathbf{C} of all valid codewords for a linear code with generator matrix \mathbf{G} and determines the minimum code distance d_{\min} .

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$$\mathbf{V} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix}, \quad \mathbf{C} = \mathbf{VG}$$

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Problem 2 continued

2. Plot the capacity of the channel as a function of γ .
 - 2.1 Use the γ from previous task.
 - 2.2 The capacity of BSC is given by the binary entropy of the channel, which again depends on error probability computed in previous task.
3. What seems to be the limit on error probability? Explain.

Problem 3

1. Hamming encoding and decoding