Signals and codes homework assignment 2019: part 1 (signals)

This document describes homework assignment in course Signals and codes, taught at Faculty of transportation sciences of Czech technical university in Prague.

1 General information

1.1 Formal requirements for submission

- The homework is assigned and collected electronically. You could bring it on paper for review or consultation during semester but to be "evaluated" it shall be **sent by an email**.
- Form of the homework is **one pdf document** of reasonable size, maximum 5 MB. Most exercises are possible to solve by hand calculation, in such case please scan respective paper(s). In case you use computational SW, copy the source code and the results. You can also combine hand and computer calculations, but please be sure that you are answering correctly the posed question(s).
- Preferably all exercises are in one document, but in case of different dates of elaboration or corrections just sent the actual (corrected / new) parts.
- Header and footer of the document shall contain information about *course, year, student name, elaboration date, page number* and *total pages*

1.2 Submission date

The deadline for submission is the end of the semester or exam date at the latest.

1.3 Scoring

Scoring is stated for each exercise separately, a total score for part 1 is 5 points.

2 Signals exercises

2.1 Signals fundamentals: power and energy [1 point]

Compute energy E, power P and effective value X_{RMS} and decide whether the signals are energy or power ones for following signals:

a) $x(t) = \begin{cases} 3 - |t| \; for \; t \in \langle -3,3 \rangle \\ 0 \; for \; other \; t \end{cases}$

b) $x[n] = 3 + \sqrt{7} \cdot j$

c) $x(t) = 325 \sin\left(314t - \frac{\pi}{4}\right)$

2.2 Spectrum of sinusoid combination [1 point]

a) A signal composed of sinusoids is given by the equation $x(t) = \sin(50\pi t + \pi/6) - 3\cos(60\pi - \pi/8)$. Sketch the spectrum of this signal indicating the complex amplitude of each frequency component. You do not have to make separate plots for real/imaginary parts or magnitude/phase. Just indicate the complex amplitude value at the appropriate frequency.

b) Is $x(t)$ periodic? If so, what is the fundamental frequency? If not, why? Which harmonics are present?

c) Now consider a different signal $y(t) = x(t) + 2\cos(25\pi t - \pi/3)$. How has the spectrum changed? Is $y(t)$ periodic? If so, what is the fundamental frequency? If not, why? Which harmonics are present?

d) Finally, consider another different signal $z(t) = x(t) + \cos(2\sqrt{5}\pi t)$. How has the spectrum changed? Is $z(t)$ periodic? If so, what is the fundamental frequency? If not, why?

2.3 Spectrum of AM modulation [1 point]

The signal $x(t)$ is formed from the signal $v(t)$ by AM modulation of carrier signal $c(t)$ according to formula $x(t) = v(t) \cdot c(t)$. Assume signals $v(t) = 4 + 2\sin(200 \cdot \pi t + \pi/6)$ and $c(t) = 2 \cdot \cos(2000 \cdot \pi t)$.

a) Find an additive combination for *x(t)*. *Hint: you can use inverse Euler formula and phasors.*
b) Count the number of frequency components in the spectrum.
c) What is the highest frequency contained in spectrum of $x(t)$?
d) Draw the spectrum for $c(t)$.
e) Draw the spectrum for $v(t)$.
f) Draw the spectrum for $x(t)$.

2.4 Fourier analysis and synthesis [1 points]

For each signal $x_1(t)$ below perform the following tasks:

- Find and plot the spectrum of $x_1(t)$ containing complex amplitudes $\{a_k\}$ of zero frequency (*DC*) and first 10 harmonics components using Fourier analysis. You can just indicate the complex amplitude value at the appropriate frequency or you can plot magnitudes of $\{a_k\}$ and phases of $\{a_k\}$ separated.
- Verify the result using Fourier synthesis, denote the synthesized signal as $x_2(t)$. Plot original signal $x_1(t)$ and synthesized one $x_2(t)$ as a proof and explain possible differences.

Assigned signals $x_1(t)$ are as follows:

a) $x_1(t) = 1 + 4\cos(100\pi t) + 3\sin(500\pi t) + 2\cos(2500\pi t - \pi/8)$

b) $x_1(t) = \begin{cases} -3 \; for \; 0 < t < \frac{T_0}{5} \\ 1 \; for \; \frac{T_0}{5} \leq t \leq T_0 \end{cases}$

c) $x_1(t)$ is measured with sample frequency 10 kHz, within one period the acquired values are

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10.38,10.25,9.00,9.14,8.10,6.81,6.30,6.04,6.15,6.12,5.55,6.87,7.59,8.00,9.28,9.57,
10.68,11.83,12.10,12.37,11.86,10.74,10.80,9.94,8.64,7.66,6.67,5.49,5.16,4.35,4.84,
4.39,5.09,5.47,6.22,7.65,8.11,8.16,8.97,7.95,7.95,7.22,6.69,5.64,3.85,2.98,1.86,
1.16,0.57,0.25,0.29,0.33,0.77,0.93,1.64,2.79,3.93,3.84,4.39,4.06,4.35,3.35,2.87,
2.18,1.41,0.53,-0.71,-1.77,-2.16,-2.15,-1.36,-1.45,-0.13,0.23,1.96,2.44,3.27,
4.15,5.11,5.29,5.22,5.47,4.77,4.12,3.79,2.46,2.33,1.91,1.36,1.64,1.55,2.21,3.60,
4.93,6.26,6.63,8.14,8.94,9.14,9.83
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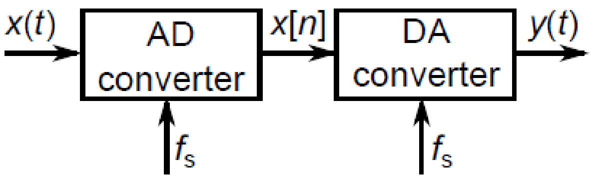
Hint: using appropriate computational SW to solve part 2.4 is recommended.

2.5 Sampling and aliasing: Waveform reconstruction [1 point]

Consider AM signal from exercise 2.3. When sampling this signal in AD converter with sample frequency f_s we get the discrete-time signal $x[n]$. When reconstructing the signal $x[n]$ in DA converter with sample frequency f_s we get the continuous-time signal $y(t)$. Assume that f_s = 2000 Hz.



a) Find expression for $x[n]$.
b) Plot spectrum of $x[n]$.
c) Find expression for $y(t)$.
d) Plot spectrum of $y(t)$.
e) What is theoretically the least possible sample frequency f_s for possibility of correct reconstruction of the signal $x(t)$?