20SK: Exercise #5

Communication channels

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November 12, 2018

with problems inspired by Proakis, Salehi, Bauch: Contemporary Communication Systems Using MATLAB®, 3rd Ed. Cenage Learning, 2011.

Introduction

Communication system objective: *Transmit information from one location to another.*

Medium: *Communication channel*.

Information source:

- Content measured by the *entropy* of source in *bits*.
- Appropriate mathematical model: *random process*.

Two channel types for this lab:

- Binary Symmetric Channel (BSC)
- Additive White Gaussian Noise Channel (AWGN)

Introduction
Channel modelling

Information-carrying signal is subject to a variety of changes:

1. *deterministic*: attenuation, distortion (linear, non-linear),
2. *probabilistic*: additive noise, multipath fading.

Deterministic is only a special case of stochastic $\Rightarrow$ mathematical modelling as stochastic dependence.

Both BSC and AWGN models have stochastic properties.

Discrete Memoryless Channel

Definition

Simplest case of stochastic dependence between input and output: $p(y|x)$, conditional probablity of receiving $y \in \mathcal{Y}$ when transmitting $x \in \mathcal{X}$.

Memoryless channel: Output at time-step i depends only on the input at time-step i.

Definition (Discrete memoryless channel, DMC)

A channel that is completely described by alphabets $\mathcal{X}$ and $\mathcal{Y}$ and the *channel transition probability matrix* $\mathbf{P}$,
$\forall x \in \mathcal{X}, y \in \mathcal{Y} : p_{ij} = p(y_j|x_i)$.

Channel capacity

In DMC case

Definition (Mutual information between two random variables)

The "amount of information" obtained about RV X through the other RV, Y:

$$I(X;Y) = \sum_{x \in X} \sum_{y \in Y} p(x,y) \log_2 \frac{p(x,y)}{p(x)p(y)}.$$

Definition (Channel capacity in general)

For channel input X and channel output Y can be computed as

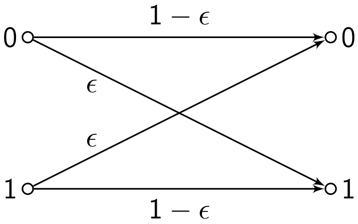
$$C = \max_{\mathbf{p}} I(X;Y).$$

Binary Symmetric Channel

A special case of DMC

Definition (Binary Symmetric Channel, BSC)

A special case of DMC, where $\mathcal{X} = \mathcal{Y} = \{0, 1\}$ and the *crossover probability* $p(y = 0|x = 1) = p(y = 1|x = 0) = \epsilon$ is symmetric.



Binary entropy: $H_\mathrm{b}(x) = -x \log_2 x - (1-x)\log_2(1-x)$.
Capacity for BSC: $C = 1 - H_\mathrm{b}(\epsilon)$.

Problem 1

1. Create function `y=bsc(x,eps)` simulating transmission of binary data from vector `x` over BSC with transition probability `eps`.
   1.1 generate a random binary input vector `x` and output vector `y`
   1.2 from `x` and `y` compute error vector `err`
   1.3 convert it to binary form, where 1 would indicate a crossover
   1.4 compare the number of errors indicated by `err` and the expected number of errors given by `eps*len(x)`
2. What type of noise distribution did we simulate in this example?
3. Does it make sense to test for $\epsilon > 0.5$? Explain.

Problem 1 continued

4. Create function `[eps,ber]=bsc_ber(n)` that will simulate the BER of BSC for packets of length `n` and different values of transition probability.

   Note 1: BER stands for *bit error ratio*, given as a ratio of number of bit errors to the total number of bits transferred.

   Note 2: BER may also denote *bit error rate*, the number of bit errors per unit time.

   What kind of function do you expect to obtain in case of BSC?

   4.1 Use `linspace` to define equally spaced crossover probability values ϵ ranging from 0.01 to 0.5.
   4.2 Use `bsc` from previous task to compute BSC output for given set of ϵ
   4.3 Comparing `x~=y` gives an vector with 1 at places where error occured
   4.4 Summing this vector gives you the number of bit errors

5. Plot the graph $\mathrm{BER} = \mathrm{bsc}(\epsilon)$ for the given packet length `n`

Additive White Gaussian Noise Channel with BPSK

Review

The channel is bandlimited to $[-W, W]$, the noise is Gaussian and white with power spectral density $N_0/2$ [W/Hz] (two-sided). Channel input satisfies power constraint P.

Definition (Capacity of AWGN)

$$C = W \log_2 \left(1 + \frac{P}{N_0 W}\right) = W \log_2 \left(1 + \frac{S}{N}\right),$$

where S/N is the signal-to-noise power ratio (watts, volts, not decibels).

Definition (Q-function)

A probability of error exceeding x is given as

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} \exp\left(-\frac{u^2}{2}\right) \mathrm{d}u = \frac{1}{2} \operatorname{erfc}\left(\frac{x}{\sqrt{2}}\right)$$

Problem 2

Binary data are transmitted over AWGN with noise power spectral density $N_0/2$ using BPSK with energy E and hard-decision decoding. In this case the channel can be modelled as BSC.

1. Plot the error probability of the channel as a function of

$$\gamma = \frac{E}{N_0}, \qquad \gamma \in [-20\,\mathrm{dB}, 20\,\mathrm{dB}].$$

   1.1 Create vector `gamma_db=-20:0.1:20` and covert it from decibels, obtaining `gamma`.
   1.2 The error probability of BPSK with optimal detection is $p = Q(\sqrt{2\gamma})$.
   1.3 Plot it using `semilogx(gamma,p)`.

Problem 2 continued

2. Plot the capacity of the channel as a function of γ.
   2.1 Use the `gamma` from task 1.1.
   2.2 The capacity of BSC is given by the binary entropy of the channel, which again depends on error probability p computed in the previous task.
3. What seems to be the limit on error probability? Explain.

Problem 3

1. Plot the capacity of AWGN channel with bandwidth $W = 3000\,\text{Hz}$ as a function of P/N_0 for values of P/N_0 between -20 and $30\,\text{dB}$.
   1.1 adopt the approach we used for `gamma`, i.e. `pn0_db=-20:0.1:30` and so on
   1.2 use the standard AWGN capacity formula
2. Now fix the P/N_0 at $25\,\text{dB}$ and plot the capacity as a function of W in the range from 10 to $10000\,\text{Hz}$.
3. Study the limit cases of capacity. Is there any difference between channel behaviour dependent on noise and bandwidth?
4. Open another figure and plot into a signle figure three bandwidth-capacity graphs for $P/N_0 = 25, 40$ and $55\,\text{dB}$. What do you observe?