Signals and its properties

Signals and codes (SK)

Department of Transport Telematics
Faculty of Transportation Sciences, CTU in Prague

Lecture 1

Lecture goal and content

Goal

- Understand what is signal, know basic types of signal and their basic characteristic values and reveal that signals are everywhere around us.

Content

- Signals
  - what is it?
  - types of signals
  - examples of signals
  - characteristic values of signals
    - instantaneous value
    - average value
    - signal energy
    - signal power
    - effective value

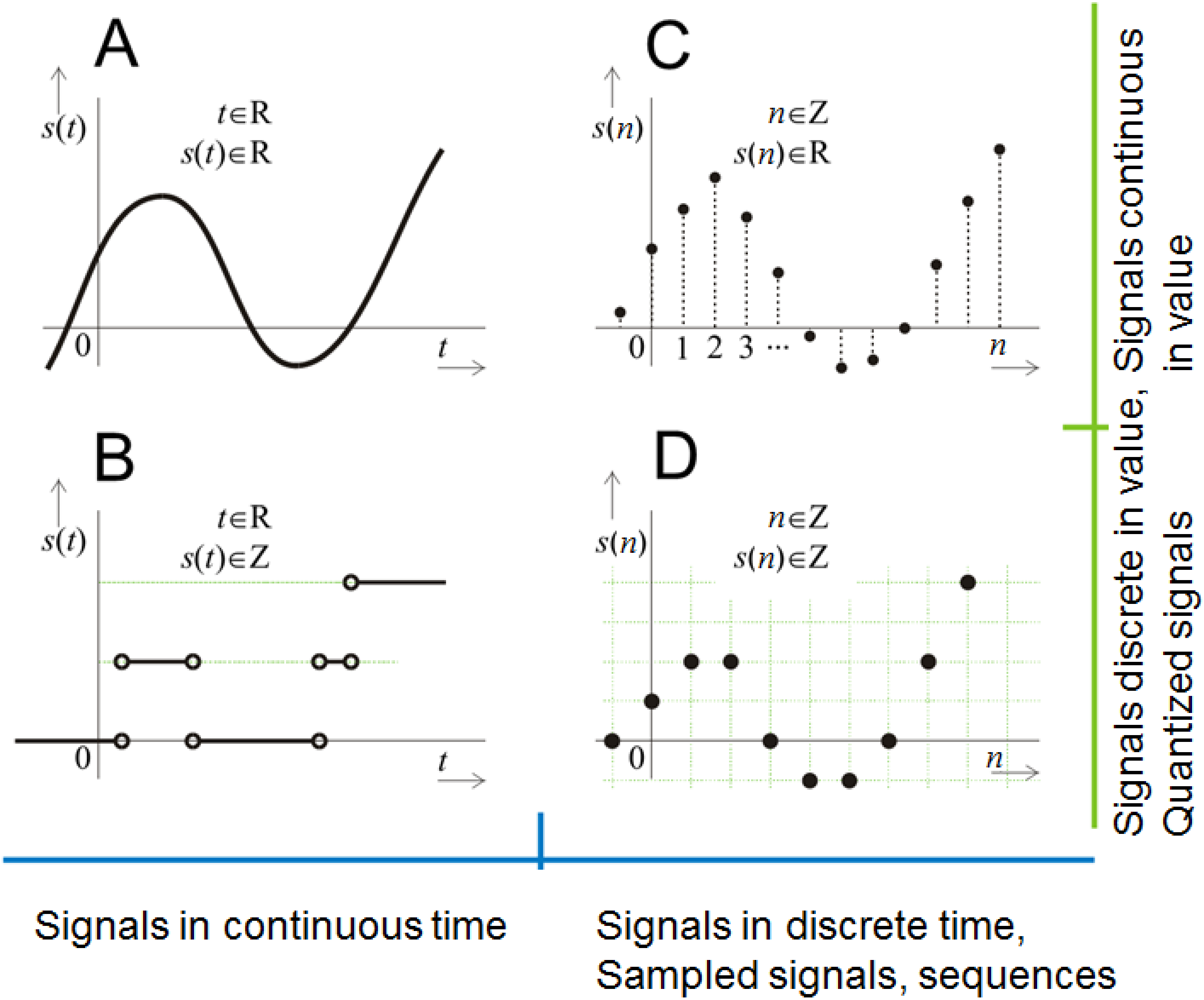
What is signal?

Definition: an abstraction of any measurable quantity that is a function of one or more independent variables such as time or space. For this course it is some function of time.

Examples:

- A voltage or a current in a circuit
- Electrocardiograms
- Sinusoid $A.\sin(\omega t + \varphi)$
- Speech/music
- Intensity of light radiation
- Acoustic pressure
- Image, video
- etc.

Signal types: continuous (C), discrete (D)

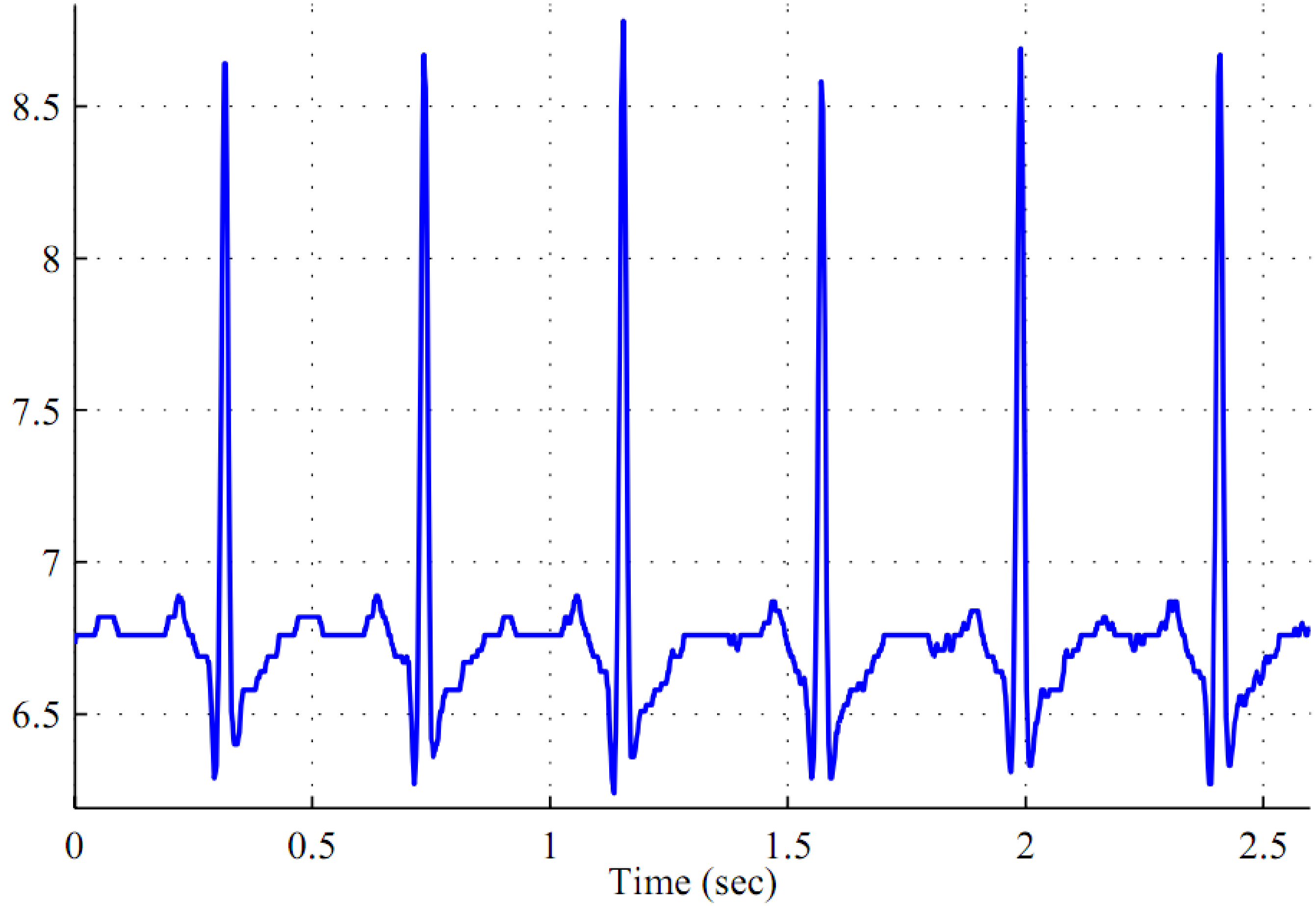


Basic signal operations

- Time shift: $s(t-t_0)$ and $s[n-n_0]$
  - If $t_0 > 0$ or $n_0 > 0$, signal is shifted to the right
  - If $t_0 < 0$ or $n_0 < 0$, signal is shifted to the left
- Time reversal: $s(-t)$ and $s[-n]$
- Time scaling: $s(at)$ and $s[an]$
  - If $a > 1$, signal is compressed
  - If $1 > a > 0$, signal is stretched
- Signal scaling: $as(t)$ and $as[n]$
  - If $a > 1$, signal has higher values
  - If $1 > a > 0$, signal has lower values

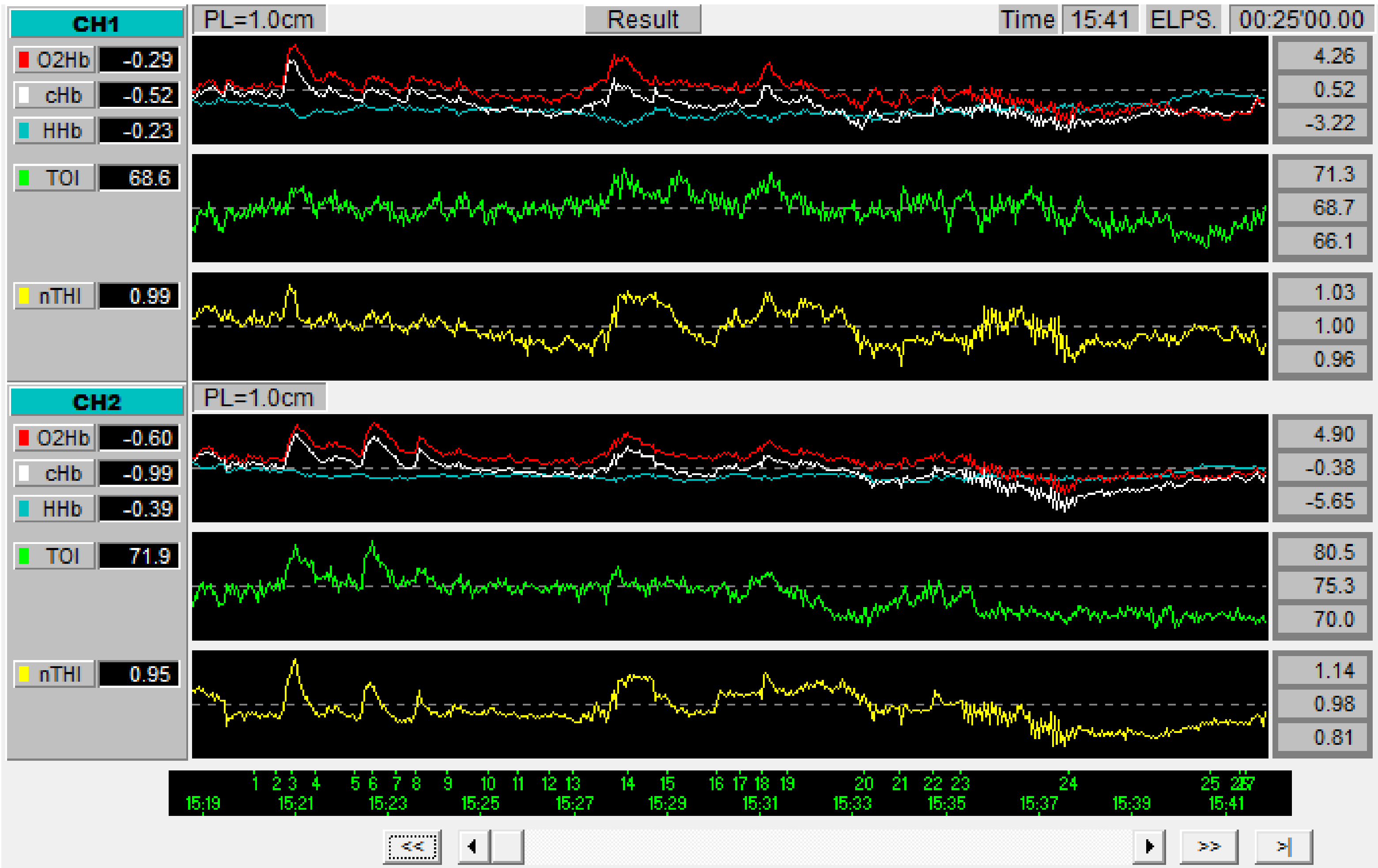
Signal examples

- ECG (electrocardiogram), in Czech EKG



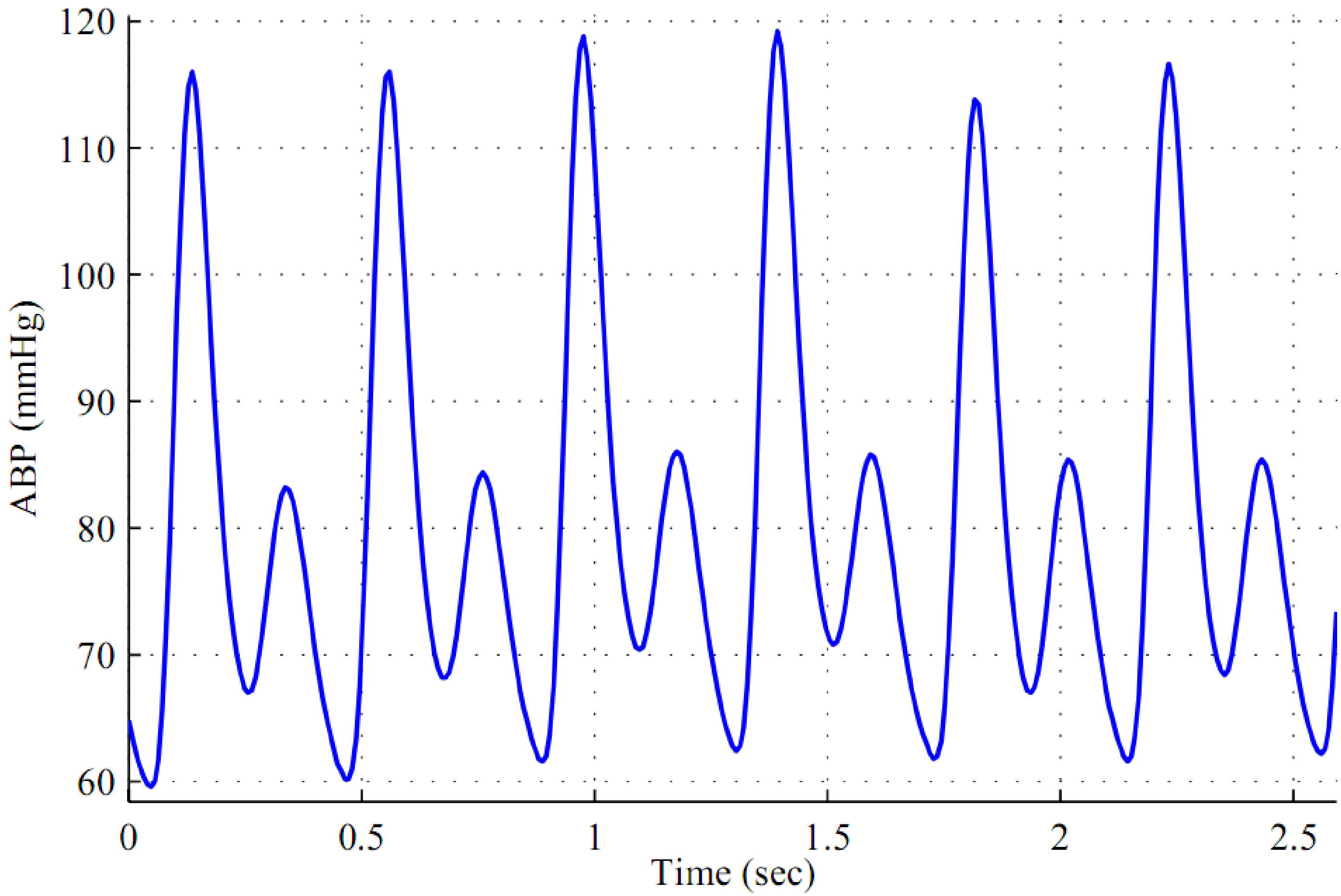
Signal examples

- EEG (electroencephalogram)



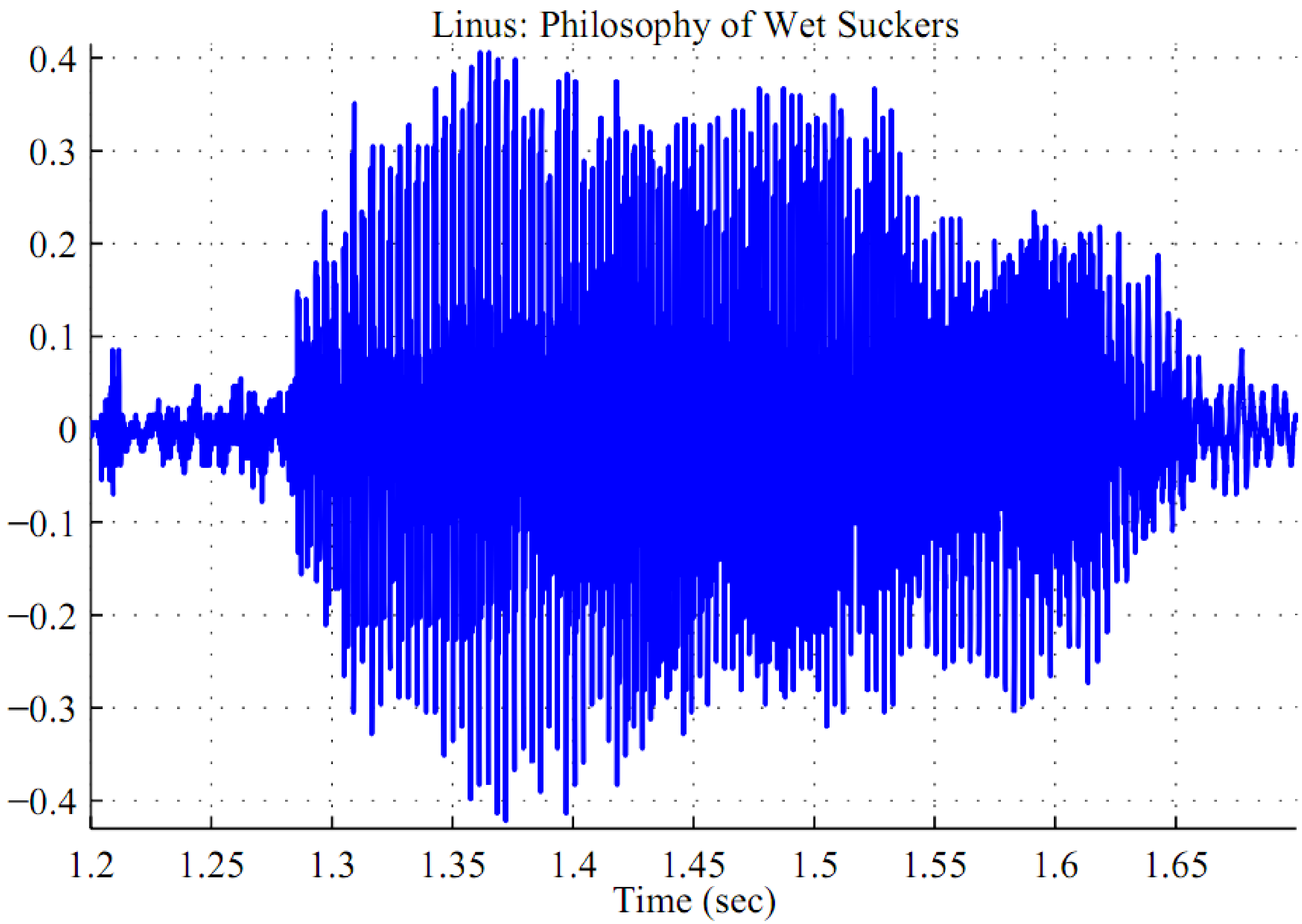
Signal examples

- Arterial pressure (tepenný tlak)



Signal examples

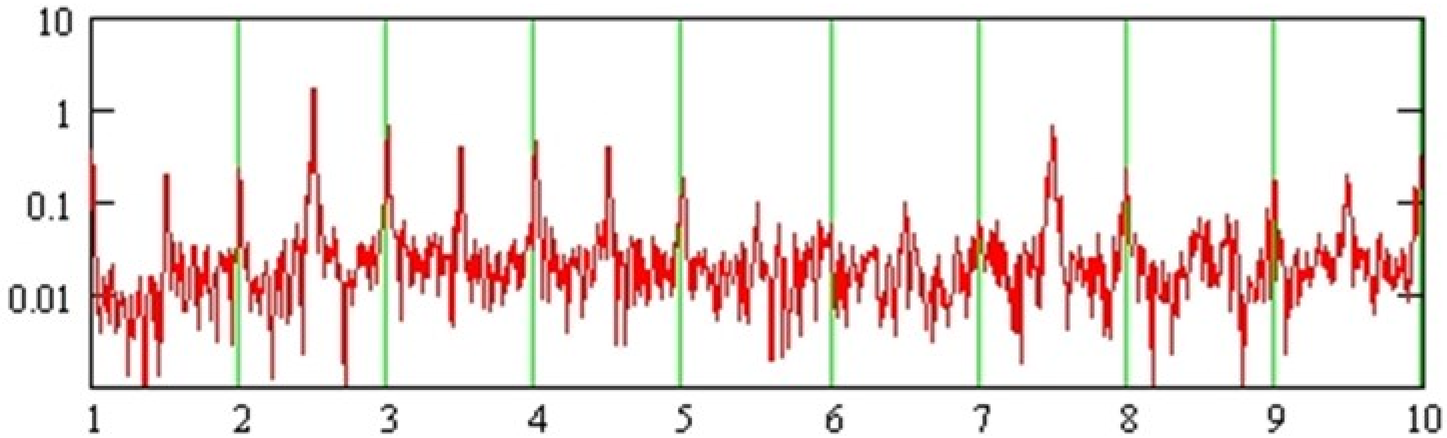
- Speech (řeč)



Signal examples

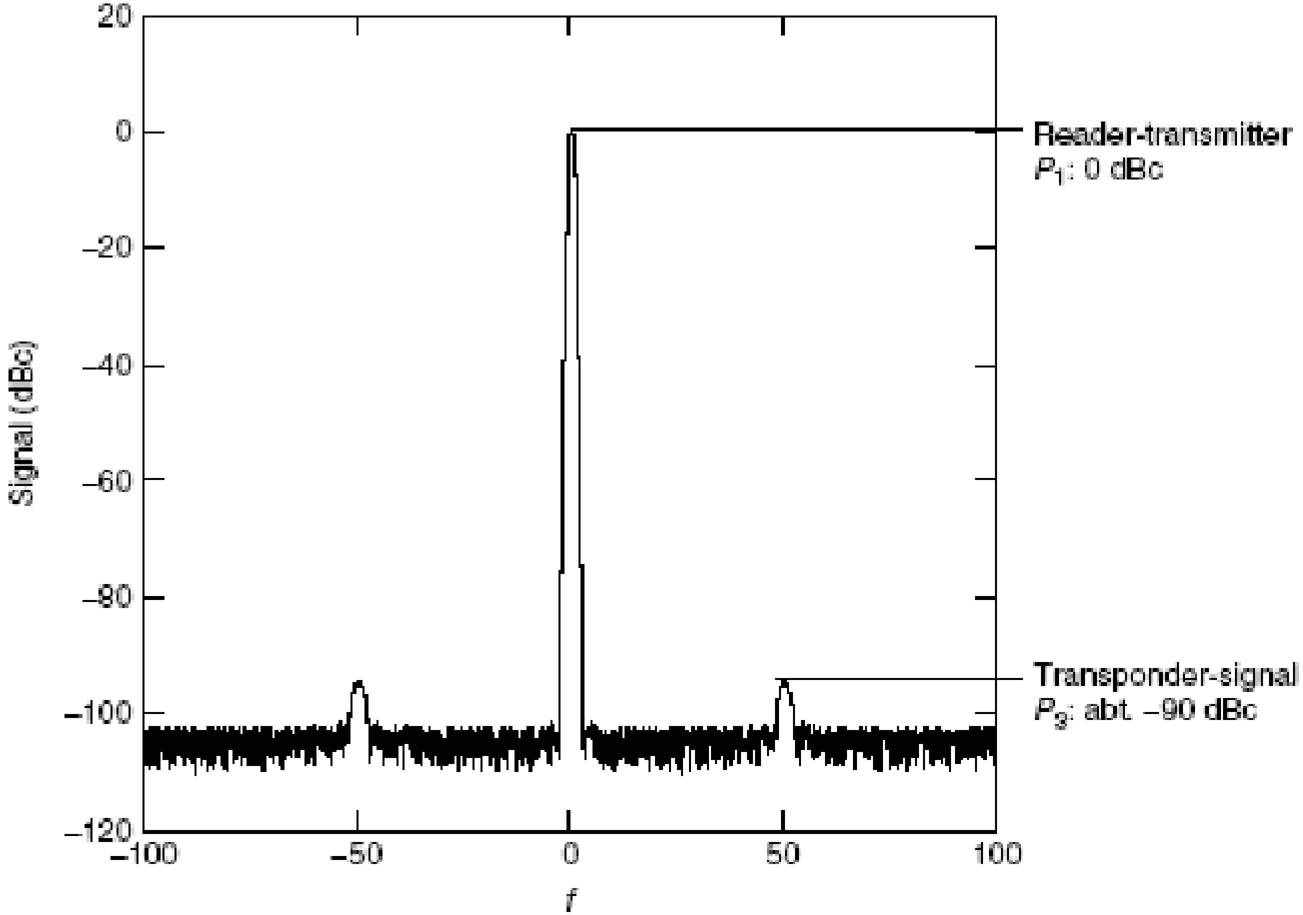
- NVH (Noise, vibration, and harshness) (hluk a vibrace)

  – Here is the noise **spectrum** of Michael Schumacher's Ferrari at 16680 rpm, showing the various harmonics. The x axis is given in terms of multiples of engine speed. The y axis is logarithmic, and uncalibrated (https://en.wikipedia.org/wiki/Noise,_vibration,_and_harshness).



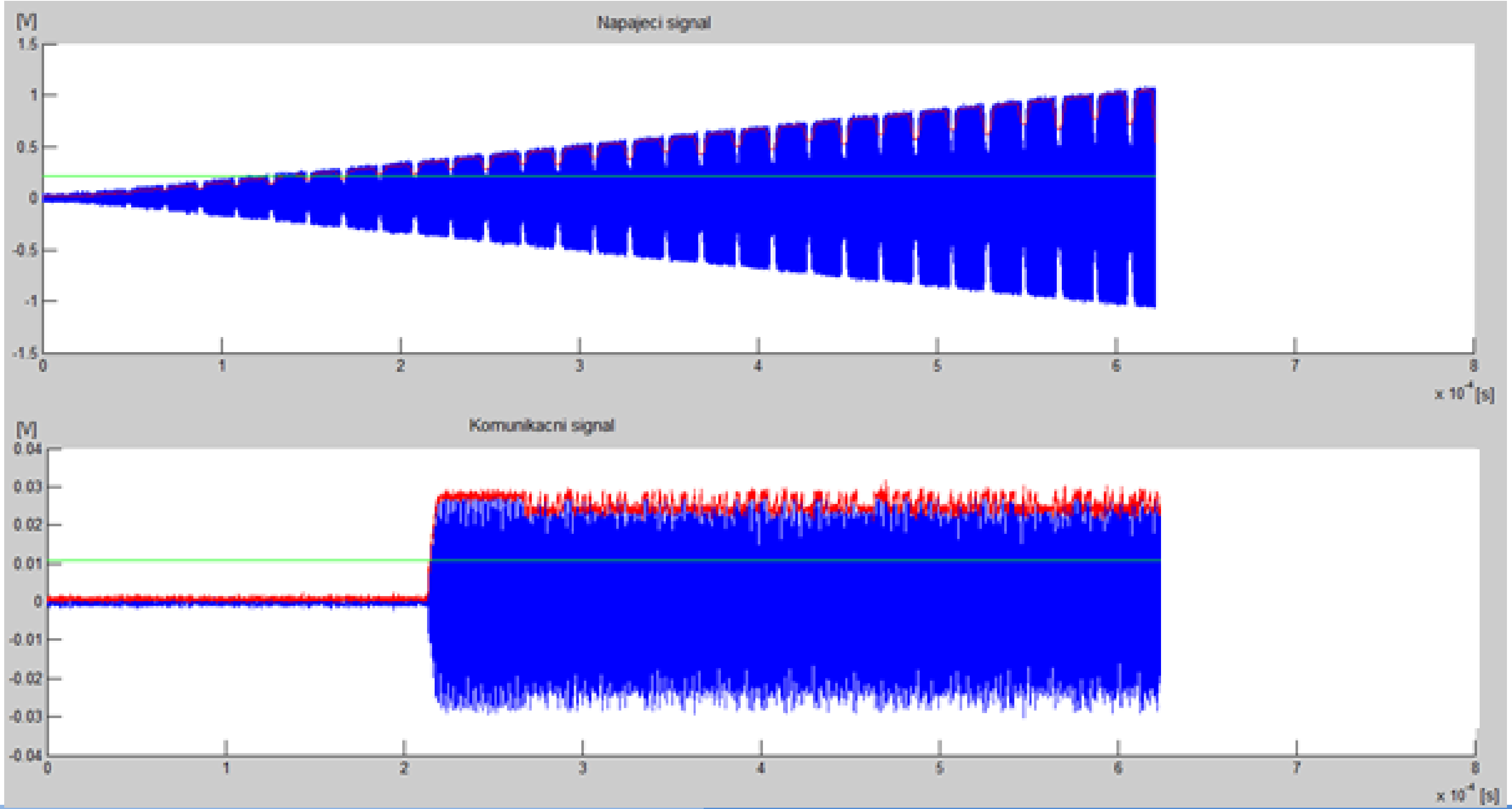
Signal examples

- RFID (not only in transport)



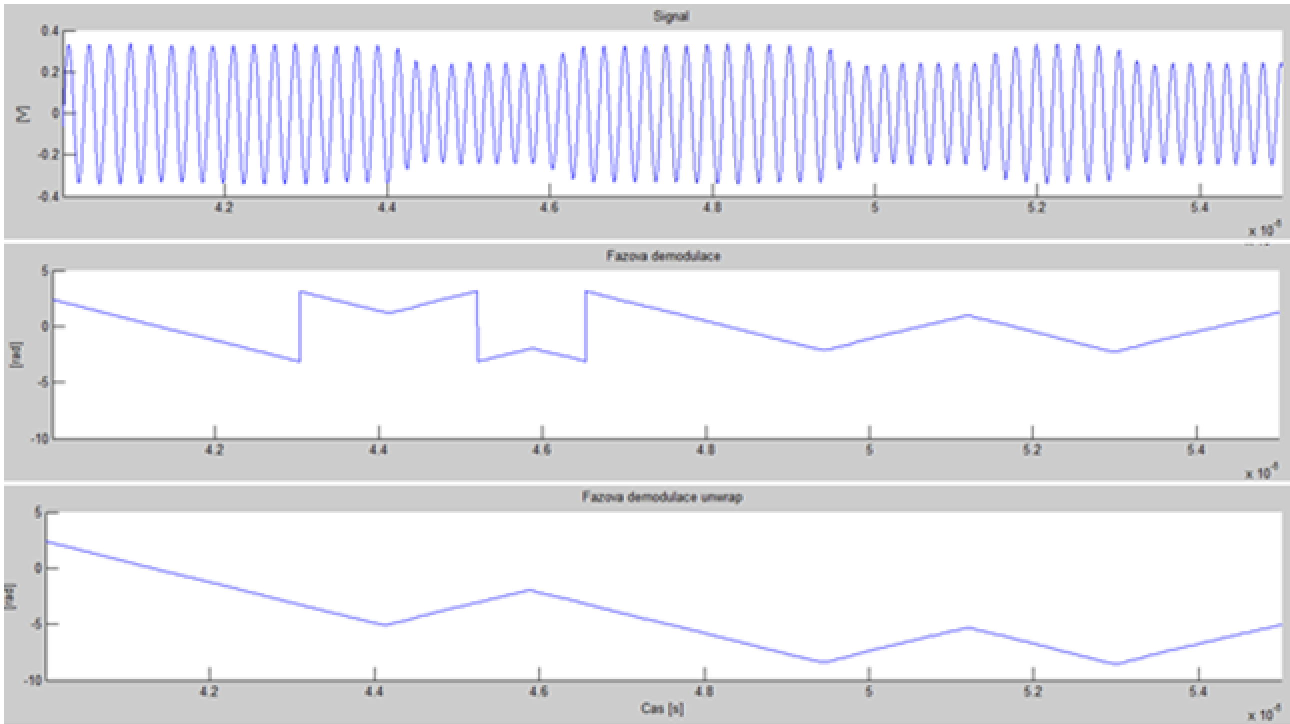
Signal examples

- Eurobalise – AM modulated tele-powering signal and up-link signal (balise responce) (napájecí a komunikační signál (odpověď balízy))

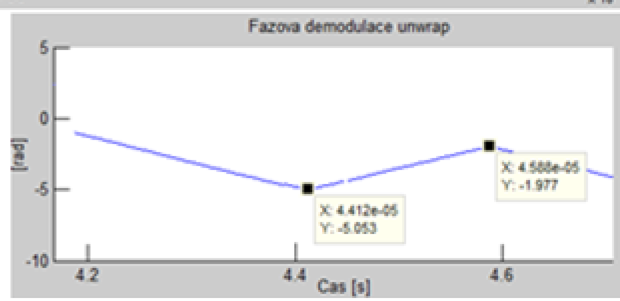


Signal examples

- Eurobalise – phase demodulation of FSK modulated up-link signal

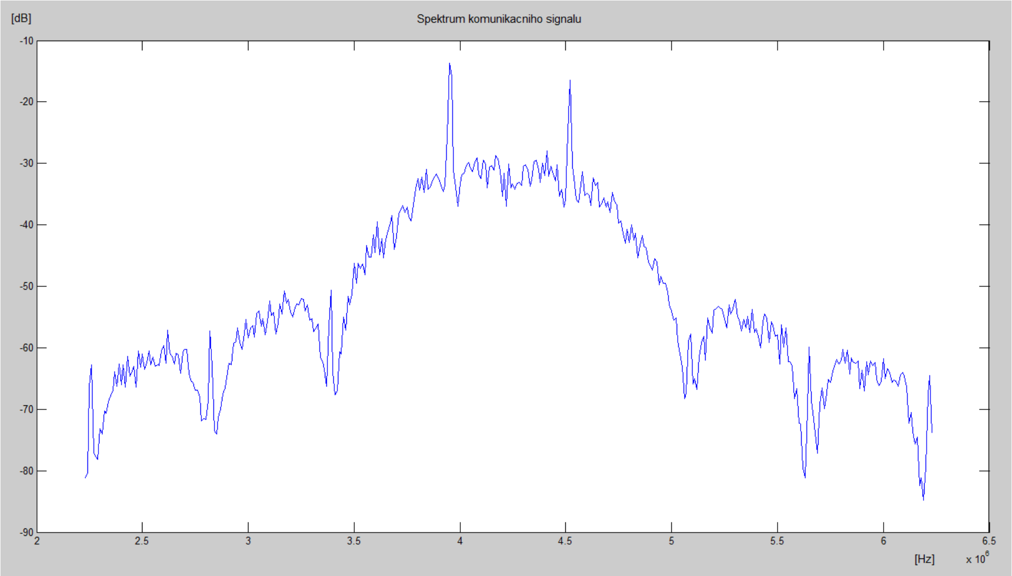


$$f = \frac{\omega_1 - \omega_2}{2.\pi.(t_1 - t_2)} + f_{\text{centre}}$$

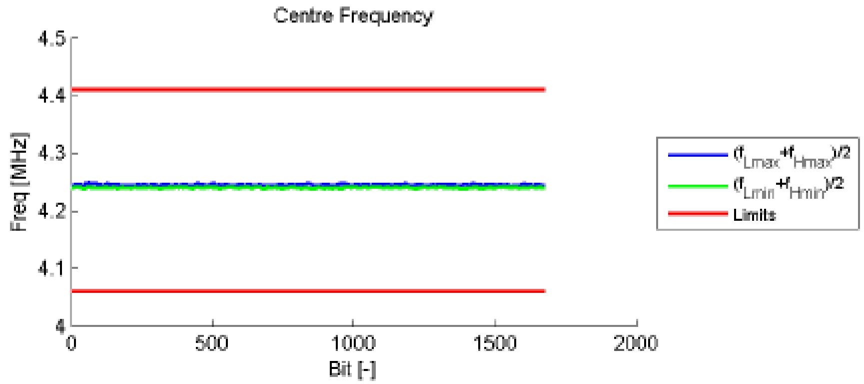


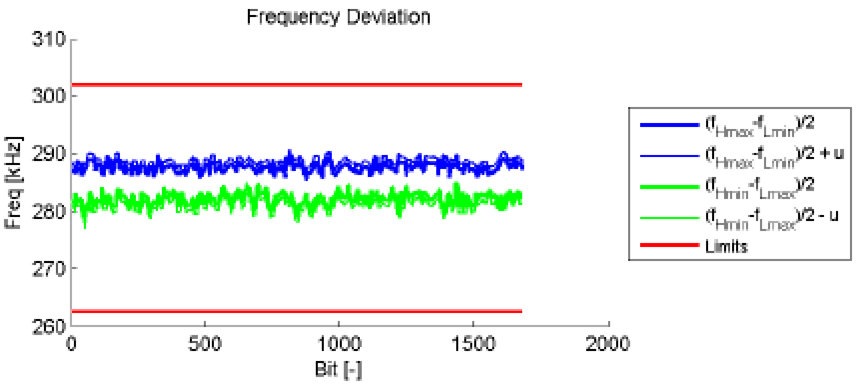
Signal examples

- Eurobalise – spectrum of up-link signal



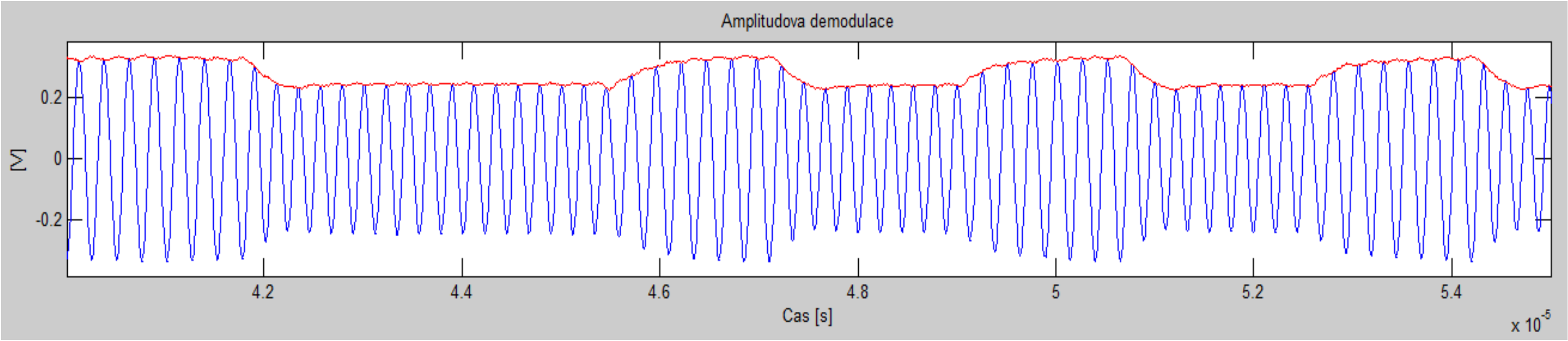
- Eurobalise – up-link signal: centre frequency and deviation





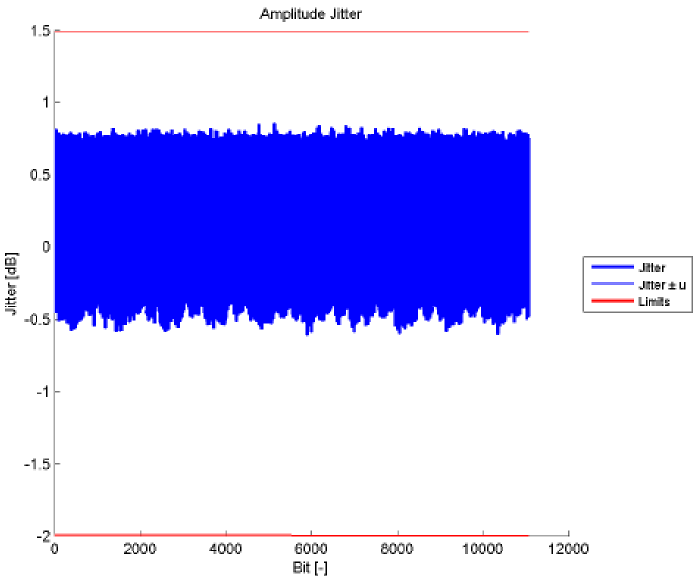
Signal examples

- Eurobalise – amplitude demodulation of up-link signal



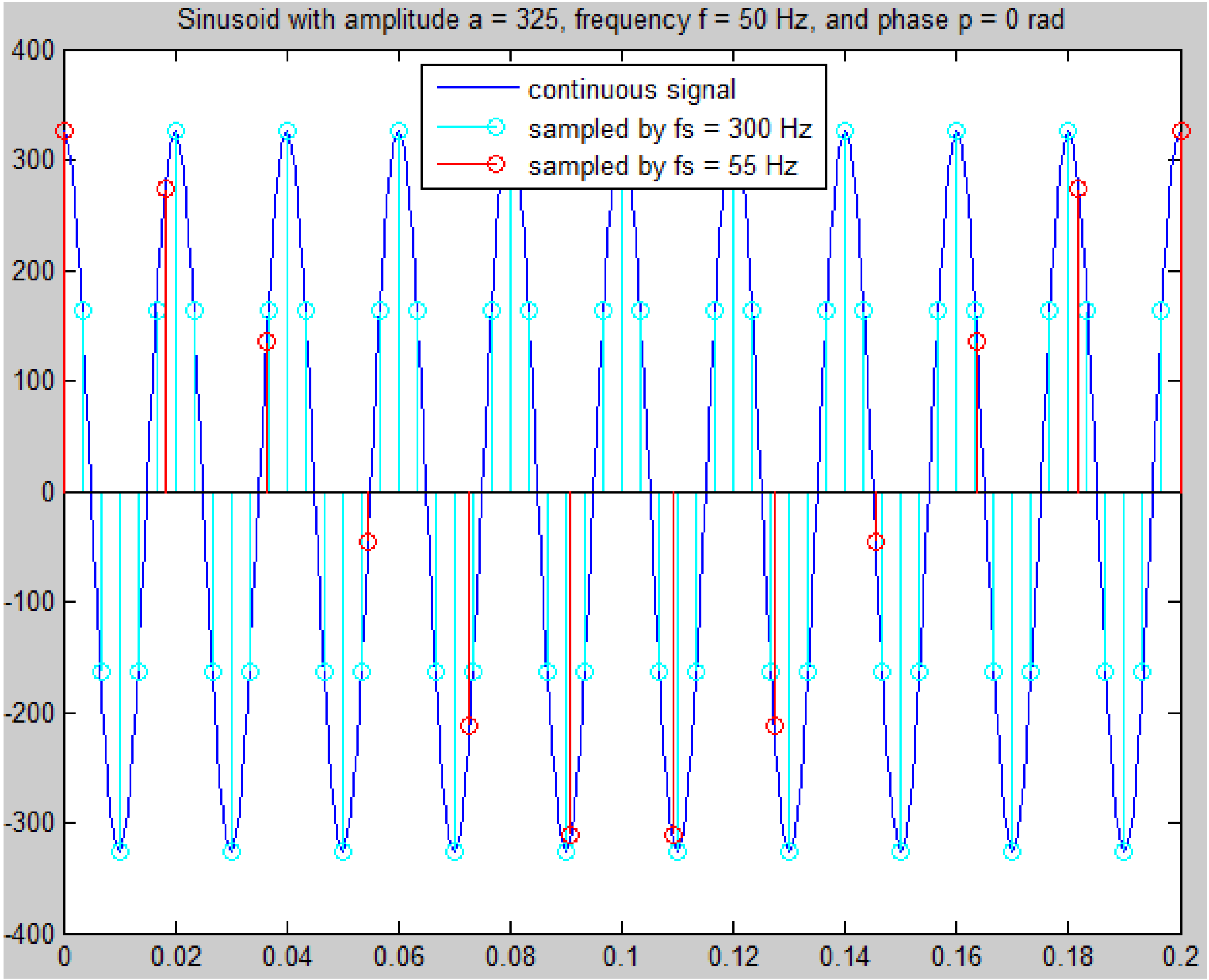
- Eurobalise – amplitude jitter

(kolísání amplitudy)



Signal examples

- Sampled socket voltage 230 V



Signal examples

- Video detection of vehicles

pictures from https://www.youtube.com/watch?v=F_M_skebbpA

Characteristic values of signals $x(t)$

① Instantaneous value $x(t_i)$, $x[n_i]$

Exe.1.1: Given signal $x(t) = 325 \cdot \sin(2\pi \cdot 50 \cdot t)$

Find instantaneous value of the signal for time instant $t = 10\,ms$.

Sol.: $\underline{x(10 \cdot 10^{-3})} = 325 \cdot \sin(2\pi \cdot 50 \cdot 0{,}01) = 325 \cdot \sin\pi = \underline{0}$ ■

Note: x is dimensionless

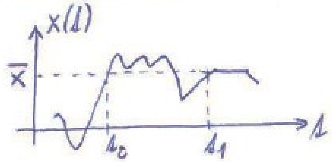
Q: Is socket voltage 230 V safe at time instant $t = 10\,ms$?

A: Yes, BUT dangerous touch takes $t > 0\,s$, see RMS value below.

② Average value $\bar{x}$ in a time interval

- Continuous time (CT):

$$\bar{x} = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} x(t)\, dt$$

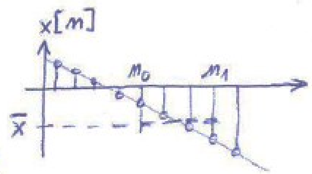


- periodic function: choose $t_1 - t_0 = T_0$ ... fundamental period

arbitrary time instant thus $\bar{x} = \frac{1}{T_0} \int_{t_i}^{t_i + T_0} x(t)\, dt$

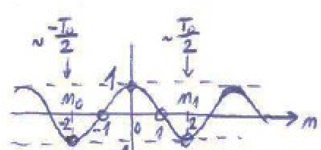
- Discrete time (DT)

$$\bar{x} = \frac{1}{m_1 - m_0 + 1} \sum_{n=m_0}^{m_1} x[n]$$



- periodic function: choose $m_1 = m_0 + \frac{T_0}{T_S} - 1$ (signal period T_0, sample period T_S)

thus: $\bar{x} = \frac{T_S}{T_0} \sum_{m_0}^{m_0 + \frac{T_0}{T_S} - 1} x[n]$

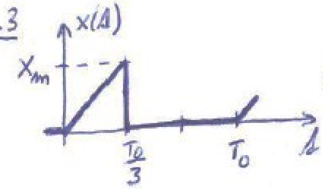


Exe.1.2: $x(t) = e^{-0{,}2t}$ Find average value $\bar{x}$ in time interval $t \in \langle 0; 2 \rangle$ s.

(Note: Transient phenomena equation ... $e^{-\frac{t}{\tau}}$, thus $\tau = 5\,s$)

Sol.: $\underline{\bar{x}} = \frac{1}{2-0} \int_0^2 e^{-0{,}2t}\, dt = \frac{1}{2}\left[\frac{-1}{0{,}2} \cdot e^{-0{,}2t}\right]_0^2 = \frac{-5}{2}\left(e^{-0{,}4} - 1\right) \doteq \underline{0{,}8242}$ ■

Exe.1.3



Find average value of the signal $x(t)$

(note: time interval = one fundamental period T_0)

(note: result is apparent at first glance)

Sol.: $\underline{\bar{x}} = \frac{1}{T_0} \int_0^{T_0} x(t)\, dt = \frac{1}{T_0} \int_0^{\frac{T_0}{3}} \frac{X_m}{\frac{T_0}{3}} \cdot t\, dt = \frac{3X_m}{T_0^2}\left[\frac{t^2}{2}\right]_0^{\frac{T_0}{3}} = \frac{3X_m}{T_0^2} \cdot \frac{T_0^2}{3^2 \cdot 2} = \underline{\frac{X_m}{6}}$

Exe.1.4: $x[n] = 325 \cdot \sin\left(2\pi \cdot 50 \cdot \boxed{\frac{1}{200}} \cdot n\right)$ $\frac{1}{f_s} = T_s$

Find average value of given discrete time periodic signal.

Sol.: $T_0 = \frac{1}{f_0} = \frac{1}{50}\,s$, $T_S = \frac{1}{200}\,s \Rightarrow \frac{T_0}{T_S} = 4$, $m_0 = 0$, $m_1 = 3$

$\underline{\bar{x}} = \frac{1}{3 - 0 + 1} \sum_{n=0}^{3} 325 \cdot \sin\left(\frac{\pi}{2} \cdot n\right) = \underline{0}$ ■

③ Signal energy E

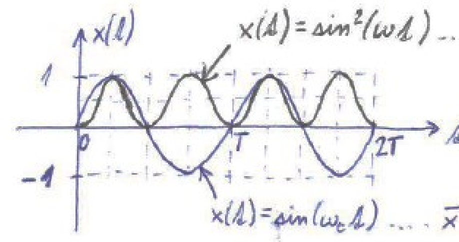
← energy signals have $0 < E < \infty$

CT: $E = \int_{-\infty}^{\infty} |x(t)|^2\, dt$

DT: $E = \sum_{n=-\infty}^{\infty} |x[n]|^2$

note: absolute value in the formulas is important for complex signals (otherwise is not necessary)

Demo:



$x(t) = \sin^2(\omega t)$ ... $\bar{x} = \frac{1}{2}$, $E = \infty$ (adding values forever, E is not proper characteristic value)

$x(t) = \sin(\omega_0 t)$ ... $\bar{x} = 0$, but there is some energy

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④ **Signal power P** ← power signals have $0 < P < \infty$

- (average) value of signal energy over a time interval (usually period). (note: physics analogy: $p = \frac{dW}{dt}$)

General CT signals:

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

General DT signals:

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} |x[n]|^2$$

Periodic CT signals:

$$P = \frac{1}{T_0} \int_{t_i}^{t_i + T_0} |x(t)|^2 \, dt$$

Periodic DT signals:

$$P = \frac{1}{N} \sum_{n = n_i + 1}^{n_i + N} |x[n]|^2, \text{ where}$$

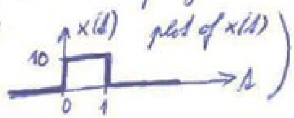
$N = \frac{T_0}{T_S}$ ... samples per period

⑤ **Effective value of a signal X_{RMS}** (root mean square)

$$X_{RMS} = \sqrt{P}$$

- equivalent constant signal with the same power as $x(t)$

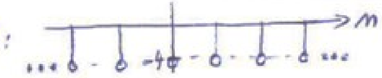
Exe. 1.5: Find energy, power and effective value of signals:

a) $x(t) = \begin{cases} 10 & \dots\ 0 \le t \le 1 \\ 0 & \dots \text{ otherwise} \end{cases}$ ( plot of $x(t)$)

Sol.: $E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = \int_0^1 10^2 \, dt = 100 \cdot [t]_0^1 = \underline{100}$

$$P = \lim_{T \to \infty} \frac{1}{2T} \boxed{\int_{-T}^{T} |x(t)|^2 \, dt} = \lim_{T \to \infty} \frac{1}{2T} \cdot 100 = \underline{0}$$

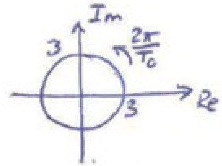
$$X_{RMS} = \sqrt{P} = \underline{0} \quad \blacksquare$$

b) $x[n] = -4 \ \forall n$ (stem plot: )

Sol.: $E = \sum_{n=-\infty}^{+\infty} |x[n]|^2 = \infty \cdot 16 = \underline{\infty}$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} |x[n]|^2 = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} |-4|^2 = \lim_{N \to \infty} \frac{2N+1}{2N+1} \cdot 16 = \underline{16}$$

$$X_{RMS} = \sqrt{P} = \underline{4} \quad \blacksquare$$

c) $x(t) = 3 \cdot e^{j \frac{2\pi}{T_0} t}$ (polar plot: )

Sol.: $E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt = 9 \cdot \infty = \underline{\infty}$

$$P = \frac{1}{T_0} \int_0^{T_0} |x(t)|^2 \, dt = \frac{1}{T_0} \int_0^{T_0} \left|3 \cdot e^{j \frac{2\pi}{T_0} t}\right|^2 dt = \frac{1}{T_0} \cdot 9 \cdot [t]_0^{T_0} = \underline{9}$$

$$X_{RMS} = \sqrt{P} = \underline{3} \quad \blacksquare$$

Energy vs. power – questions and answers

Q: Is some energy signal also power signal?
A: No. If $0 < E < \infty$, then $P = 0$. If $0 < P < \infty$, then $E = \infty$.

Q: Are all signals either energy or power signals?
A: No. Any infinite duration increasing amplitude signal will not be either.
(example: $x(t) = t^2$ is neither power signal ($P = \infty$) nor energy signal ($E = \infty$).)

Vocabulary:
- instantaneous value – okamžitá hodnota
- time instant – časový okamžik
- average value – průměrná hodnota
- fundamental period – základní perioda
- arbitrary – libovolný
- transient phenomenon – přechodný děj

01-2

References

- Vejražka, František. Signály a soustavy / 4.vyd. Praha: ČVUT, 1996. 243 s. ISBN 80-01-00450-3., In Czech