

Signals and its properties

Signals and codes (SK)

Department of Transport Telematics
Faculty of Transportation Sciences, CTU in Prague

Lecture 1



Lecture goal and content

Goal

- Understand what is signal, know basic types of signal and their basic characteristic values and reveal that signals are everywhere around us.

Content

- Signals
 - what is it?
 - types of signals
 - examples of signals
 - characteristic values of signals
 - instantaneous value
 - average value
 - signal energy
 - signal power
 - effective value

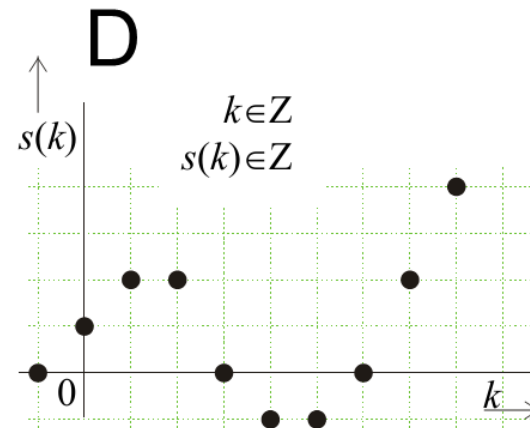
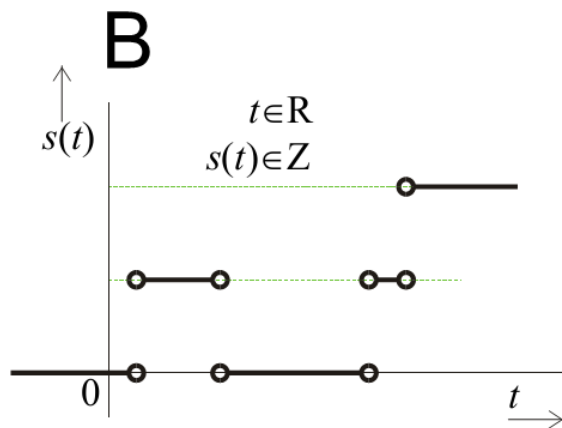
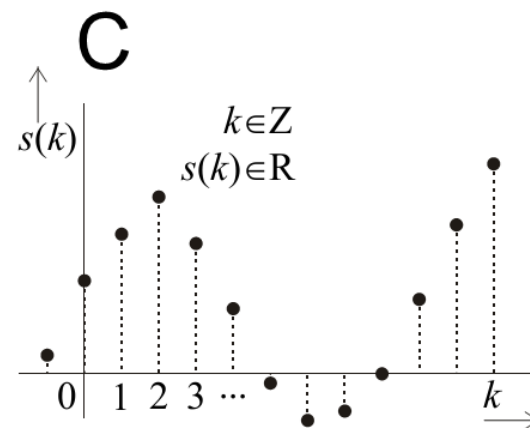
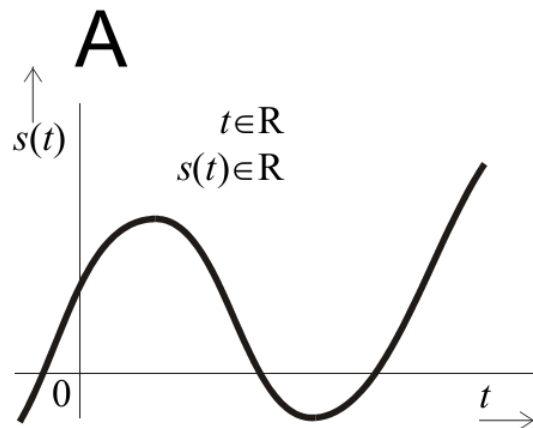
What is signal?

Definition: an abstraction of any measurable quantity that is a function of one or more independent variables such as time or space. For this course it is some function of time.

Examples:

- A voltage or a current in a circuit
- Electrocardiograms
- Sinusoid $A \cdot \sin(\omega t + \varphi)$
- Speech/music
- Intensity of light radiation
- Acoustic pressure
- Image, video
- etc.

Signal types: continuous (C), discrete (D)



Signals continuous
in value

Signals discrete in value,
Quantized signals

Signals in continuous time

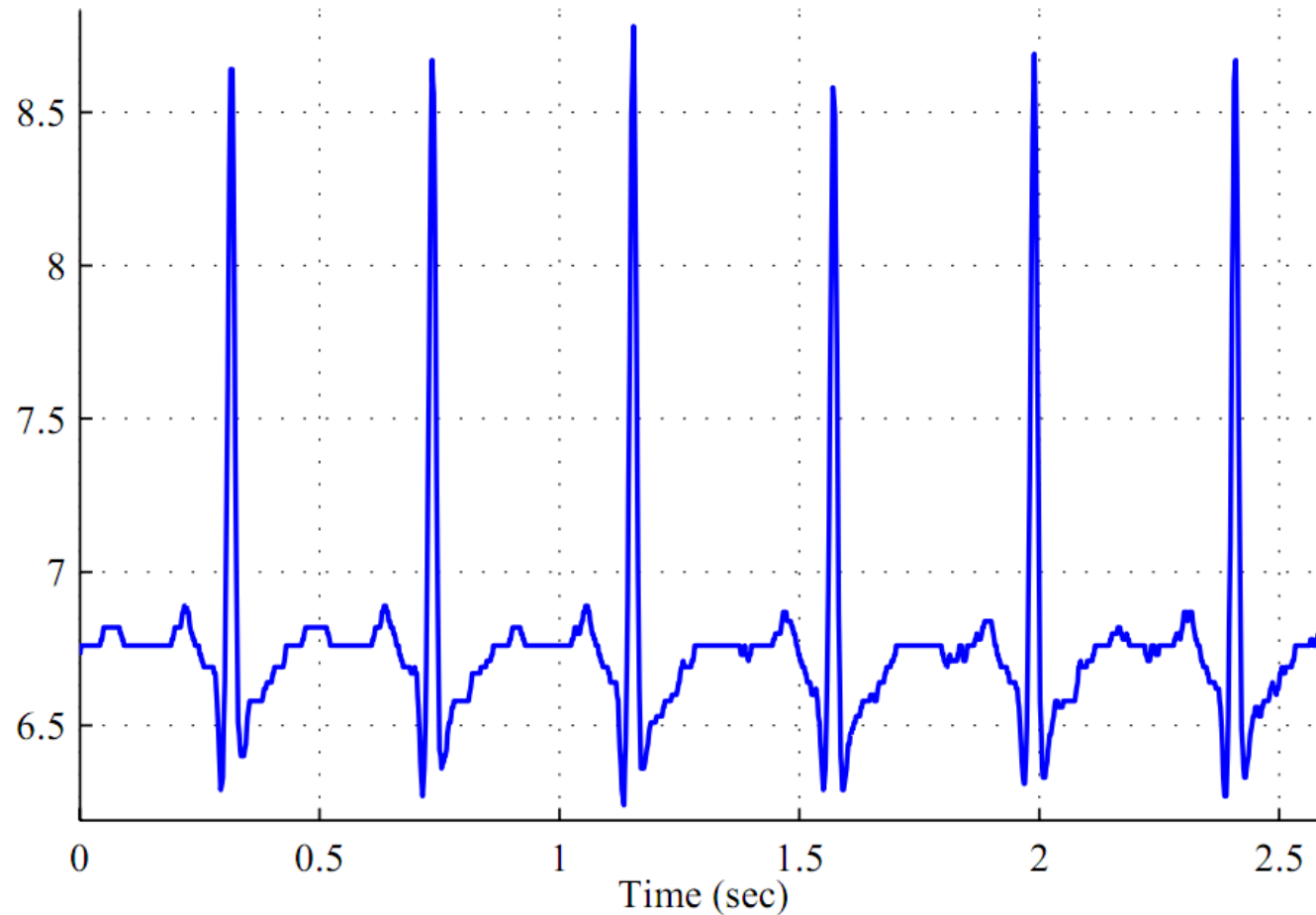
Signals in discrete time,
Sampled signals, sequences

Basic signal operations

- Time shift: $s(t-t_0)$ and $s[k-k_0]$
 - If $t_0 > 0$ or $k_0 > 0$, signal is shifted to the right
 - If $t_0 < 0$ or $k_0 < 0$, signal is shifted to the left
- Time reversal: $s(-t)$ and $s[-k]$
- Time scaling: $s(at)$ and $s[ak]$
 - If $a > 1$, signal is compressed
 - If $1 > a > 0$, signal is stretched
- Signal scaling: $as(t)$ and $as[k]$
 - If $a > 1$, signal has higher values
 - If $1 > a > 0$, signal has lower values

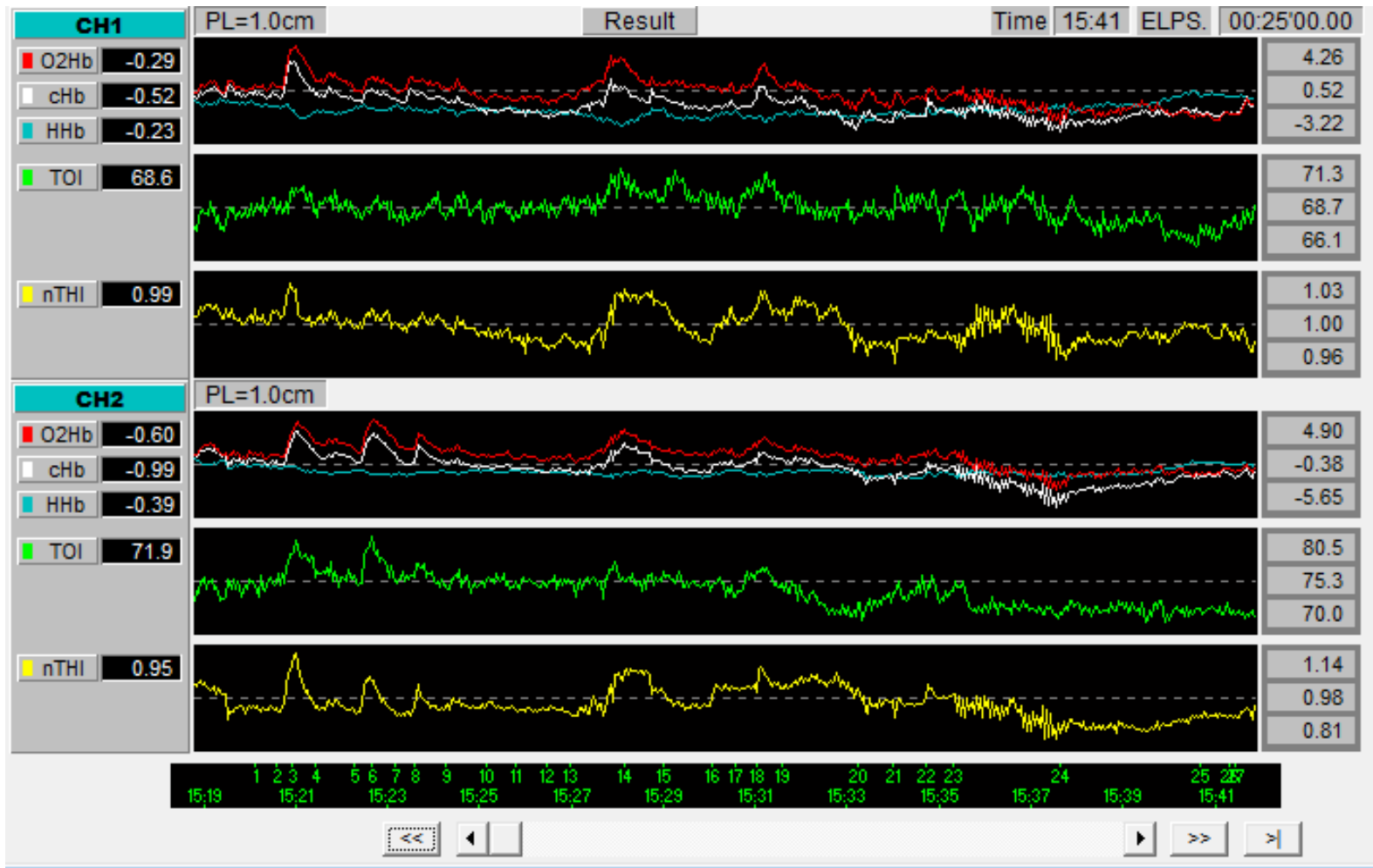
Signal examples

- ECG (electrocardiogram), in Czech EKG



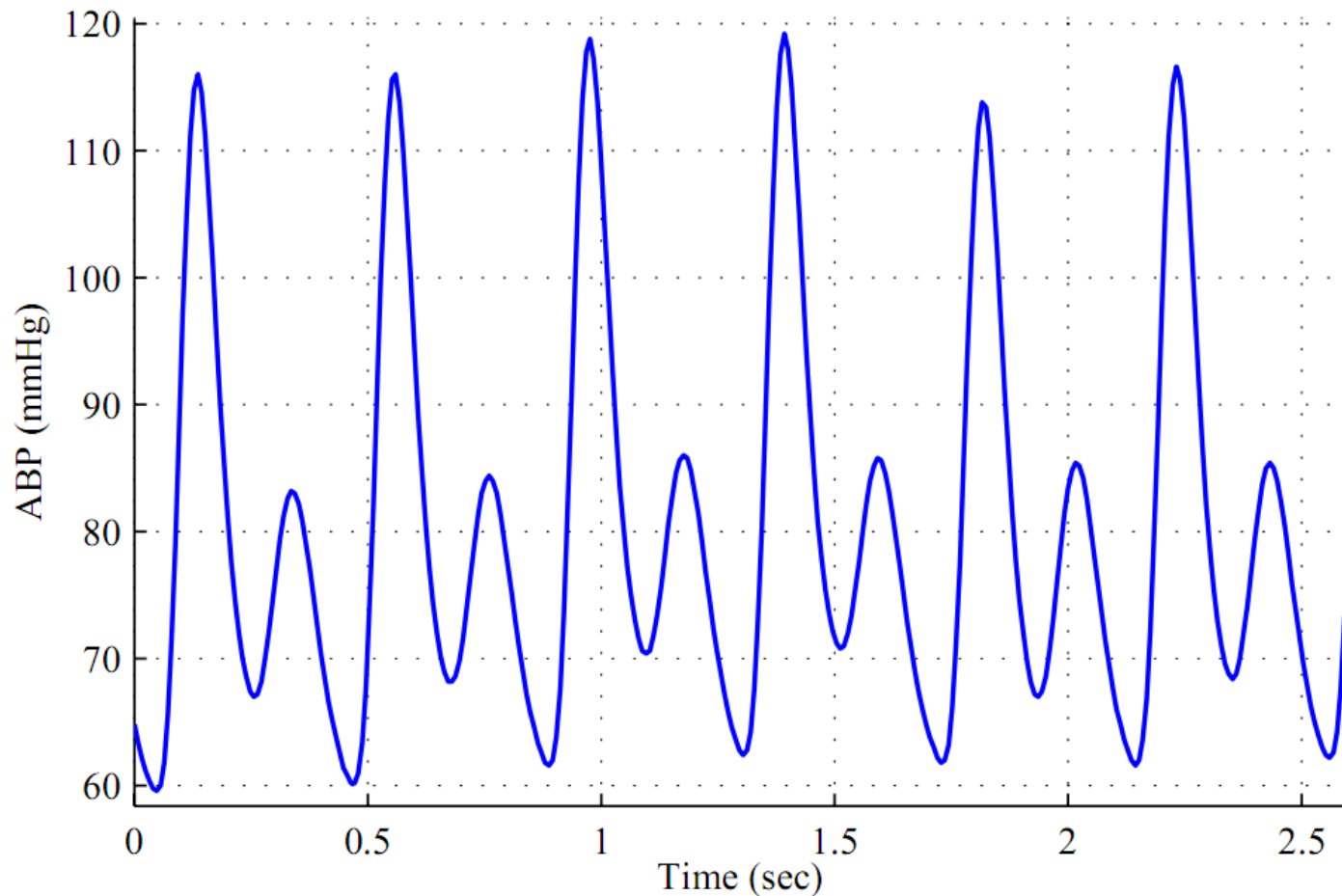
Signal examples

- EEG (electroencephalogram)



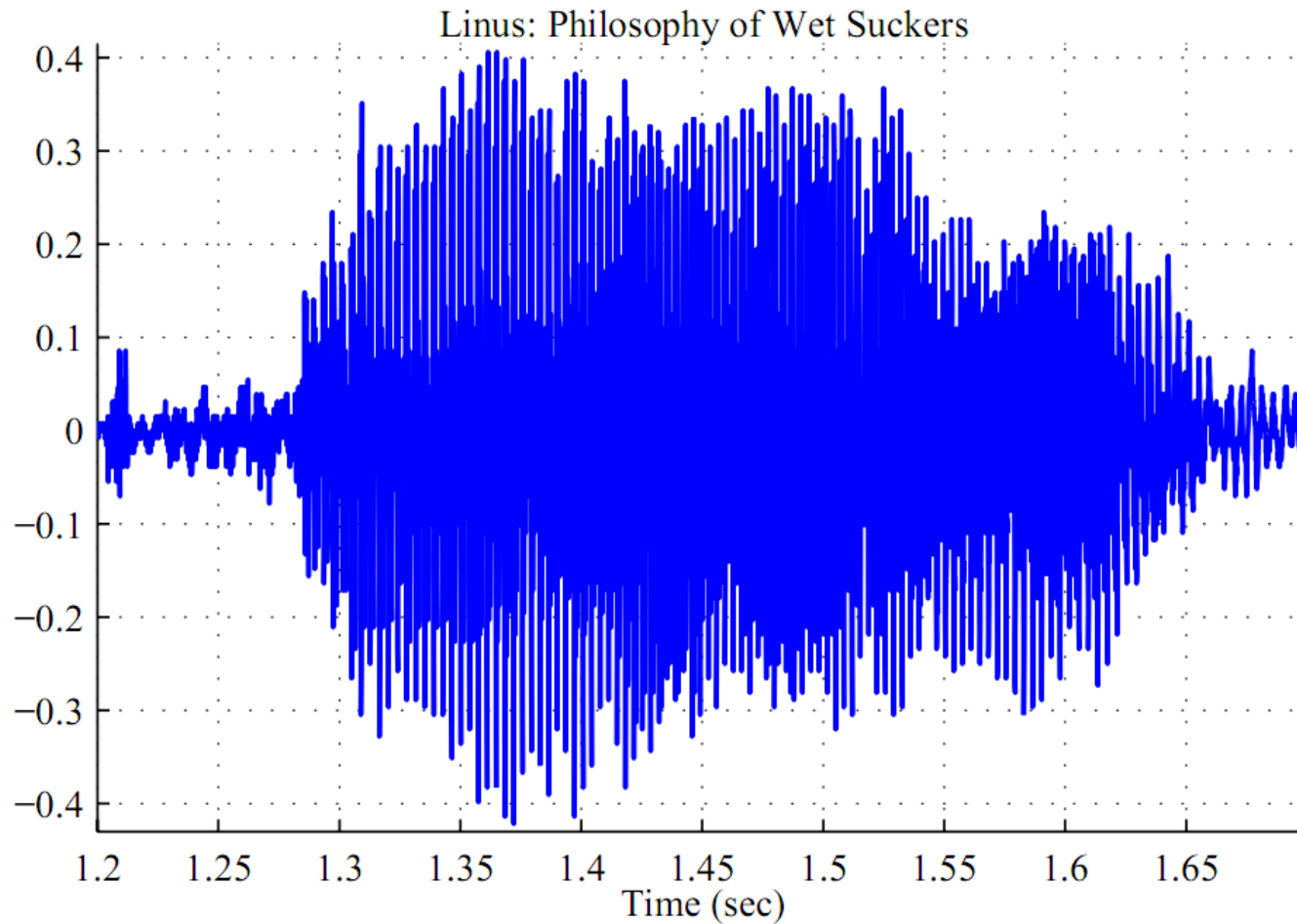
Signal examples

- Arterial pressure (tepenný tlak)



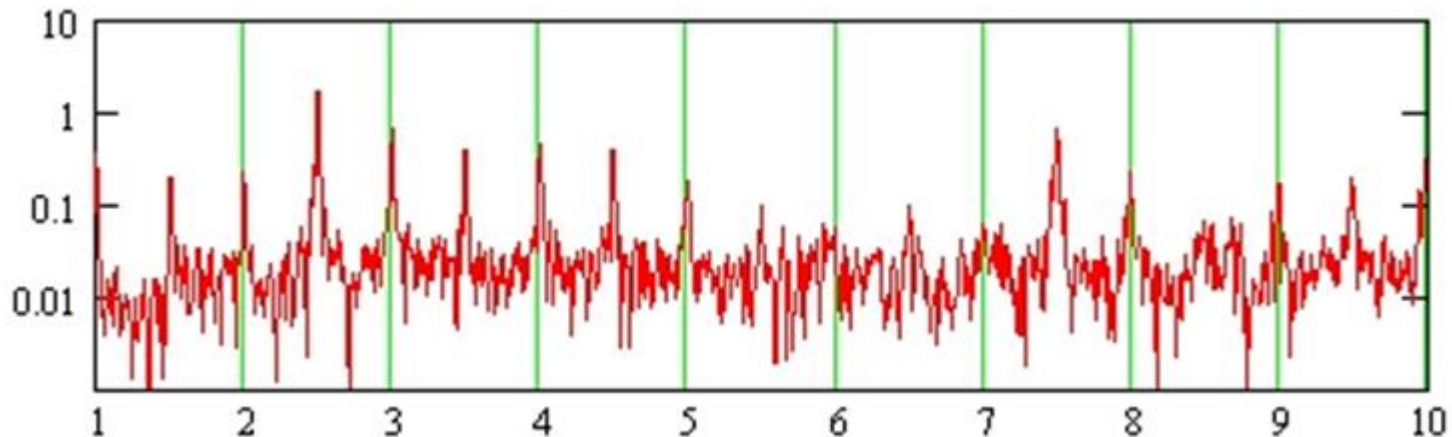
Signal examples

- Speech (řeč)



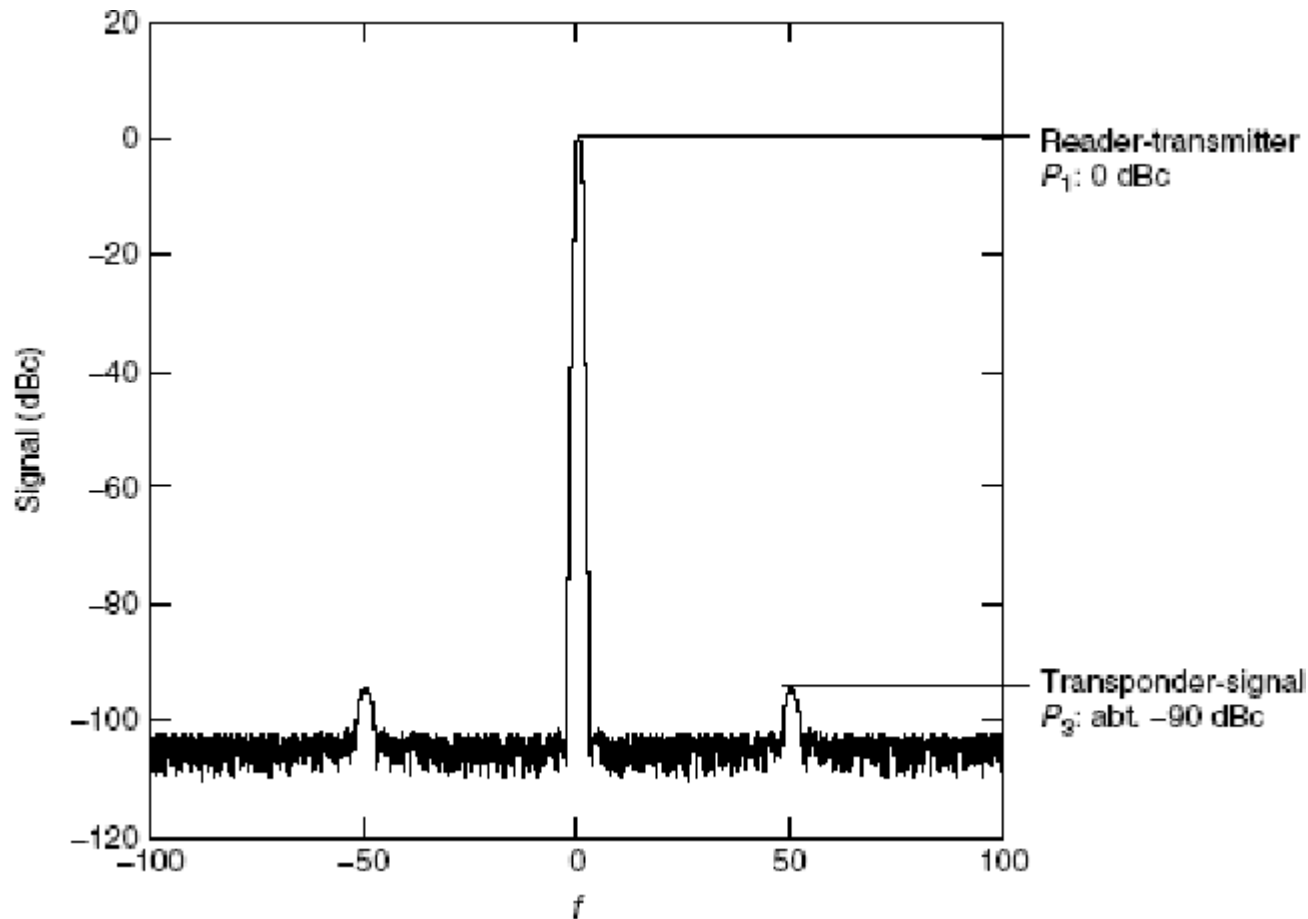
Signal examples

- NVH (Noise, vibration, and harshness) (hluk a vibrace)
 - Here is the noise **spectrum** of Michael Schumacher's Ferrari at 16680 rpm, showing the various harmonics. The x axis is given in terms of multiples of engine speed. The y axis is logarithmic, and uncalibrated (https://en.wikipedia.org/wiki/Noise,_vibration,_and_harshness).



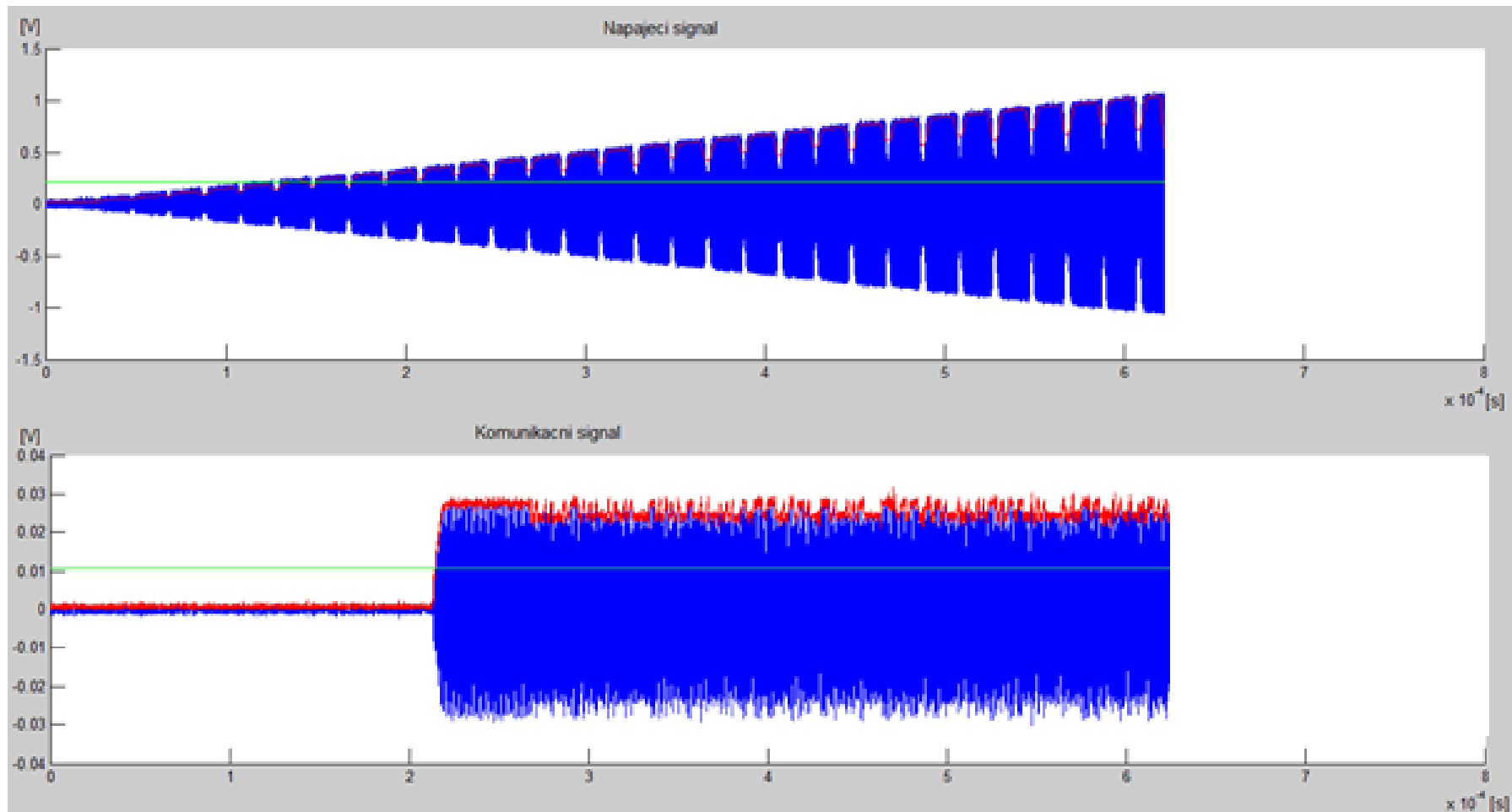
Signal examples

- RFID (not only in transport)



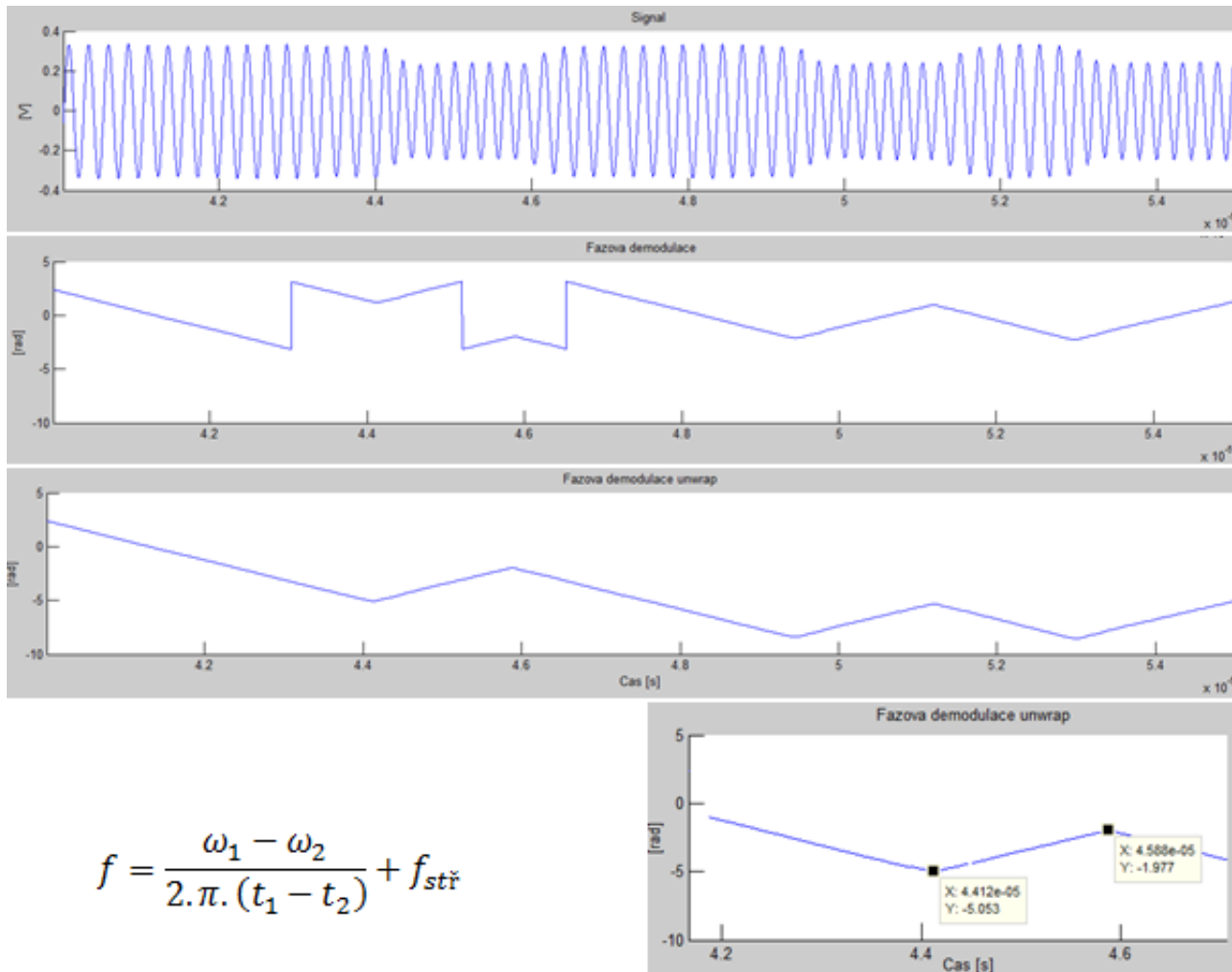
Signal examples

- Eurobalise – AM modulated tele-powering signal and up-link signal (balise response) (napájecí a komunikační signál (odpověď balízy))



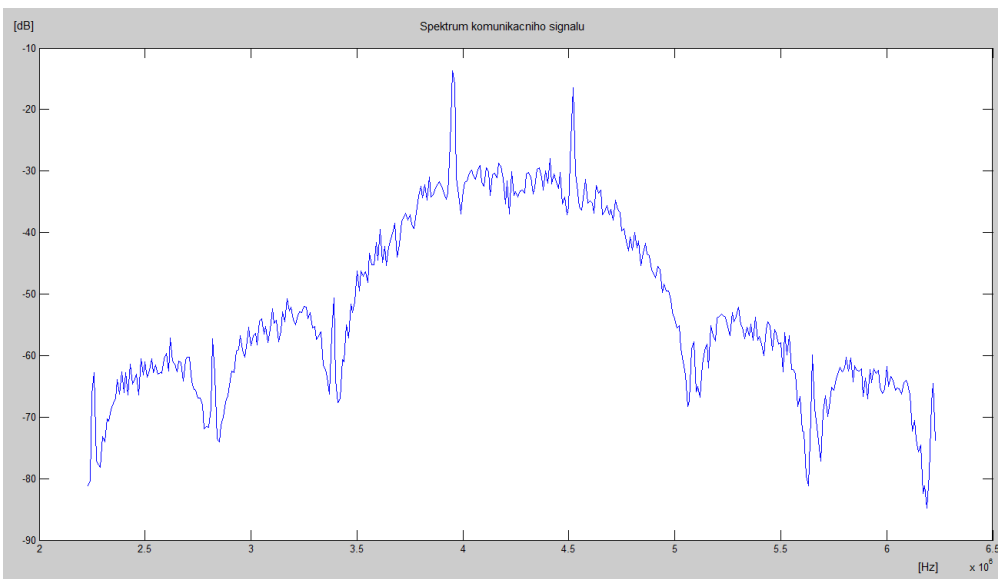
Signal examples

- Eurobalise – phase demodulation of FSK modulated up-link signal

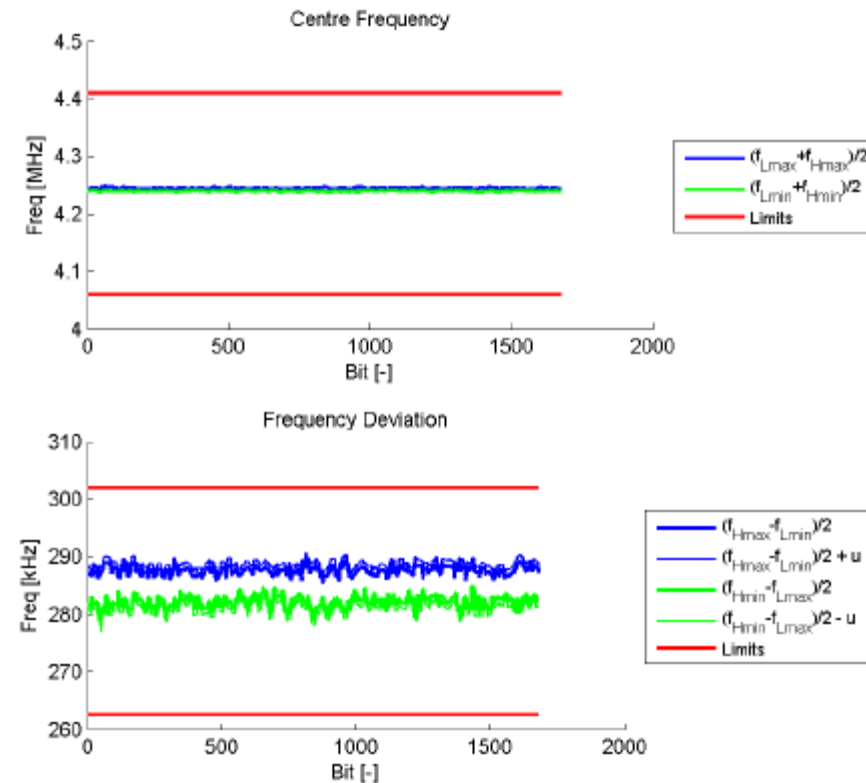


Signal examples

- Eurobalise – spectrum of up-link signal

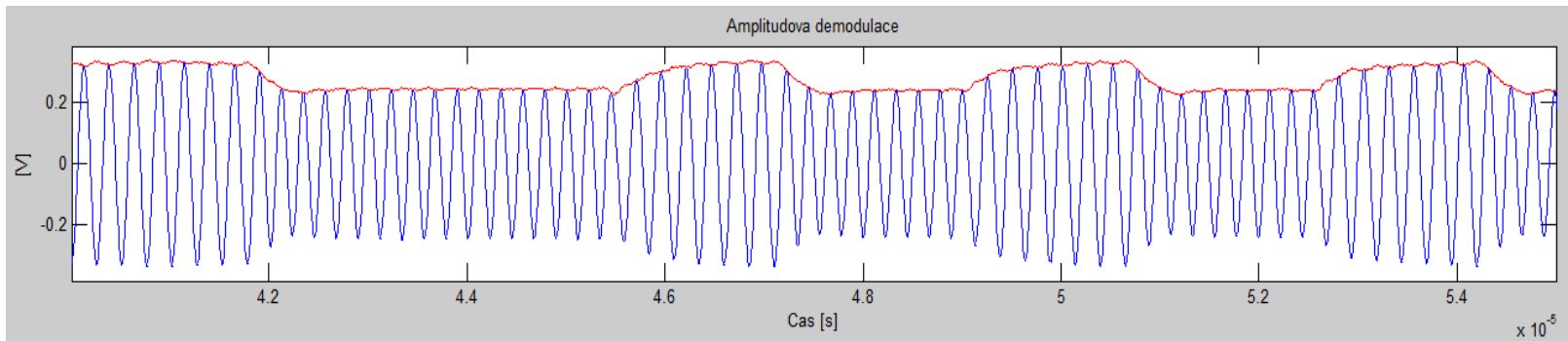


- Eurobalise – up-link signal: centre frequency and deviation

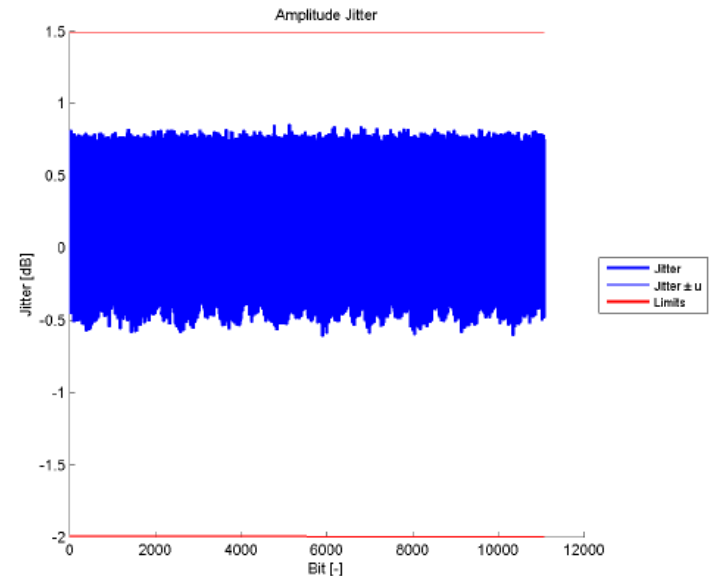


Signal examples

- Eurobalise – amplitude demodulation of up-link signal

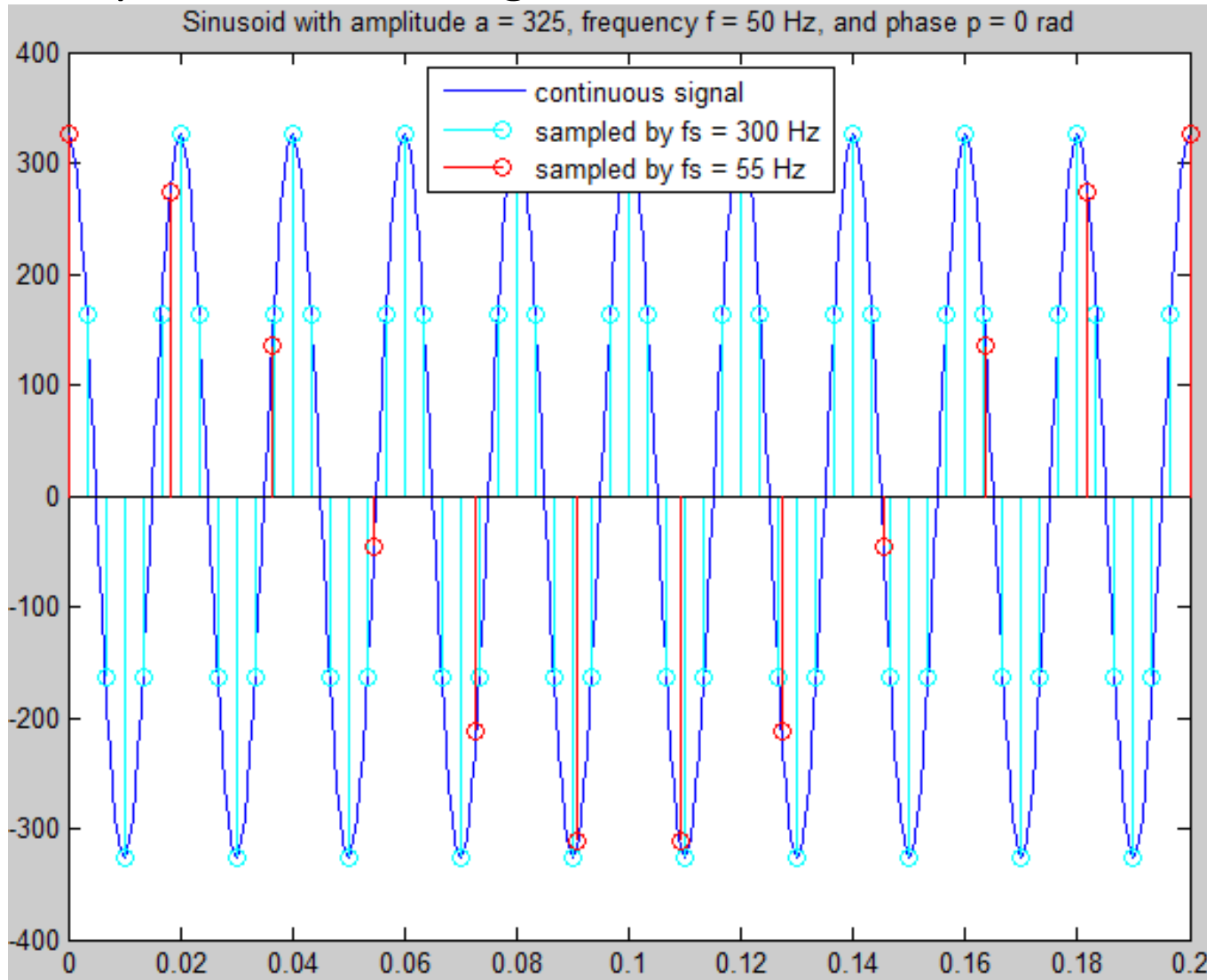


- Eurobalise – amplitude jitter
(kolísání amplitudy)



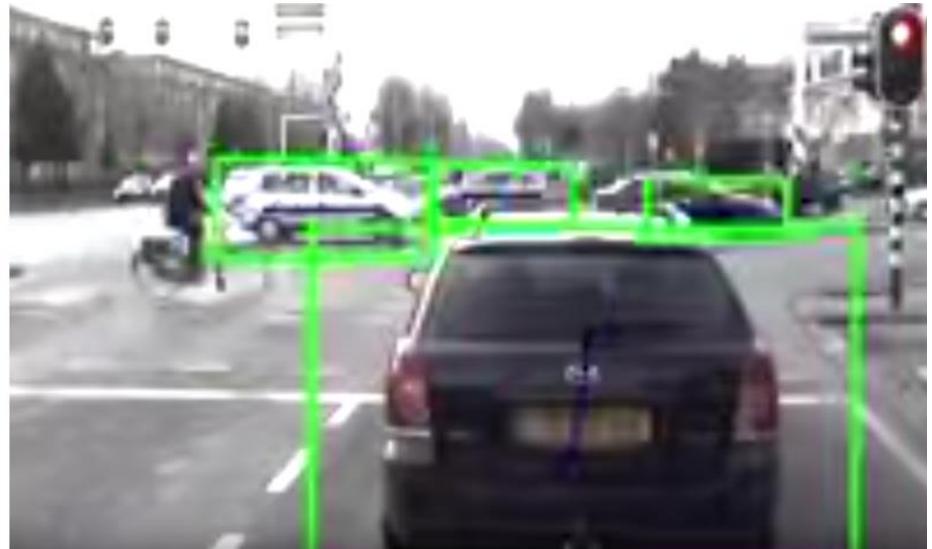
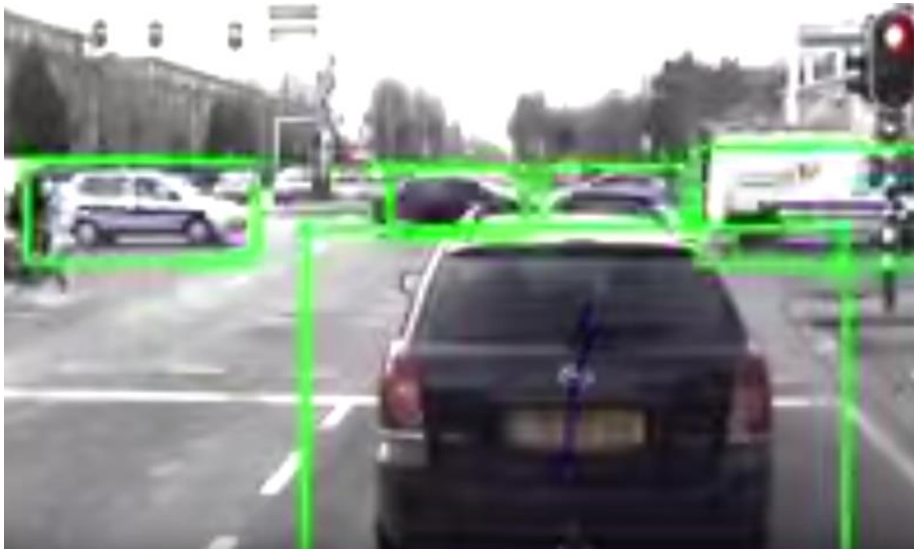
Signal examples

- Sampled socket voltage 230 V



Signal examples

- Video detection of vehicles



pictures from https://www.youtube.com/watch?v=F_M_skebbpA

Characteristic values of signals $x(t)$

① Instantaneous value $x(t_i)$, $x[n_i]$

Exe. 1.1: Given signal $x(t) = 325 \cdot \sin(2\pi \cdot 50t)$

Find instantaneous value of the signal for time instant $t = 10 \text{ ms}$.

Sol.: $x(10 \cdot 10^{-3}) = 325 \cdot \sin(2\pi \cdot 50 \cdot 0.01) = 325 \cdot \sin\pi = \underline{0}$

Note: x is dimensionless

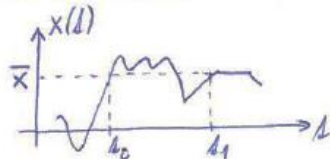
Q: Is socket voltage 230 V safe at time instant $t = 10 \text{ ms}$?

A: Yes, BUT dangerous touch takes $t > 0 \text{ s}$, see RMS value below.

② Average value $\bar{x}$ in a time interval

- Continuous time (CT):

$$\bar{x} = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} x(t) dt$$

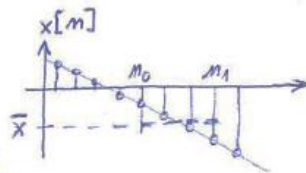


- periodic function: choose $t_1 - t_0 = T_0 \dots$ fundamental period

arbitrary time instant thus $\bar{x} = \frac{1}{T_0} \int_{t_i}^{t_i + T_0} x(t) dt$

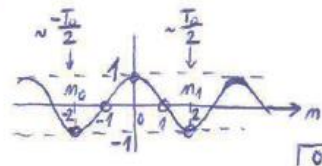
- Discrete time (DT)

$$\bar{x} = \frac{1}{m_1 - m_0 + 1} \sum_{n=m_0}^{m_1} x[n]$$



- periodic function: choose $m_1 = m_0 + \frac{T_0}{T_s} - 1$

thus: $\bar{x} = \frac{T_s}{T_0} \sum_{n=m_0}^{m_0 + \frac{T_0}{T_s} - 1} x[n]$

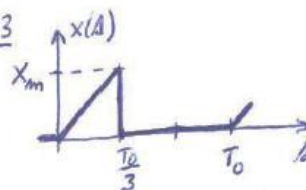


Exe. 1.2: $x(t) = e^{-0.2t}$

Find average value $\bar{x}$ in time interval $t \in (0; 2) \text{ s}$.
(Note: Transient phenomena equation $\dots e^{-\frac{t}{\tau}}$, thus $\tau = 5 \text{ s}$)

Sol.: $\bar{x} = \frac{1}{2-0} \int_0^2 e^{-0.2t} dt = \frac{1}{2} \left[\frac{-1}{0.2} e^{-0.2t} \right]_0^2 = \frac{-5}{2} (e^{-0.4} - 1) = \underline{0.8242}$

Exe. 1.3



Find average value of the signal $x(t)$
(note: time interval = one fundamental period T_0)
(note: result is apparent at first glance)

Sol.: $\bar{x} = \frac{1}{T_0} \int_0^{T_0} x(t) dt = \frac{1}{T_0} \int_0^{T_0/3} \frac{X_m}{T_0/3} t dt = \frac{3X_m}{T_0^2} \left[\frac{t^2}{2} \right]_0^{T_0/3} = \frac{3X_m}{T_0^2} \cdot \frac{T_0^2}{3^2 \cdot 2} = \underline{\frac{X_m}{6}}$

Exe. 1.4: $x[n] = 325 \cdot \sin(2\pi \cdot 50 \cdot \frac{1}{200} \cdot n)$ $\frac{1}{T_s} = T_s$

Find average value of given discrete time periodic signal.

Sol.: $T_0 = \frac{1}{f_0} = \frac{1}{50} \text{ s}$, $T_s = \frac{1}{200} \text{ s} \Rightarrow \frac{T_0}{T_s} = 4$, $m_0 = 0$, $m_1 = 3$

$\bar{x} = \frac{1}{3-0+1} \sum_{n=0}^3 325 \cdot \sin(\frac{\pi}{2} \cdot n) = \underline{0}$

③ Signal energy E

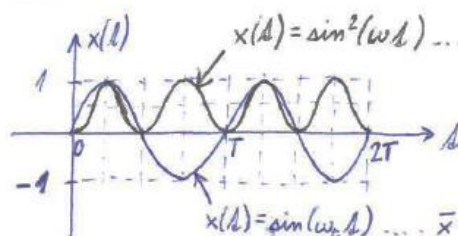
← energy signals have $0 < E < \infty$

CT: $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

note: absolute value in the formulas is important for complex signals (otherwise is not necessary)

DT: $E = \sum_{n=-\infty}^{\infty} |x[n]|^2$

Demo:



$x(t) = \sin^2(\omega t) \dots \bar{x} = \frac{1}{2}$, $E = \infty$ (adding values forever, E is not proper characteristic value)

$x(t) = \sin(\omega t) \dots \bar{x} = 0$, but there is some energy

④ Signal power P ← power signals have $0 < P < \infty$
 - (average) value of signal energy over a time interval (usually period).
 (note: physics analogy: $p = \frac{dW}{dt}$)

General CT signals:

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt$$

General DT signals:

$$P = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N |x[n]|^2$$

Periodic CT signals:

$$P = \frac{1}{T_0} \int_{t_0}^{t_0+T_0} |x(t)|^2 dt$$

Periodic DT signals:

$$P = \frac{1}{N} \sum_{m=m_1}^{m_1+N} |x[n]|^2, \text{ where}$$

$$N = \frac{T_0}{T_s} \dots \text{samples per period}$$

⑤ Effective value of a signal X_{RMS} (root mean square)

$$X_{RMS} = \sqrt{P}$$

- equivalent constant signal with the same power as $x(t)$

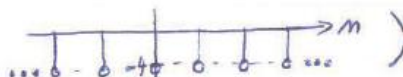
Exe. 1.5: Find energy, power and effective value of signals:

a) $x(t) = \begin{cases} 10 & \dots 0 \leq t \leq 1 \\ 0 & \dots \text{otherwise} \end{cases}$ (plot of $x(t)$)

Sol.:
$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_0^1 10^2 dt = 100 \cdot [t]_0^1 = 100$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt = \lim_{T \rightarrow \infty} \frac{1}{2T} \cdot 100 = 0$$

$$X_{RMS} = \sqrt{P} = 0$$

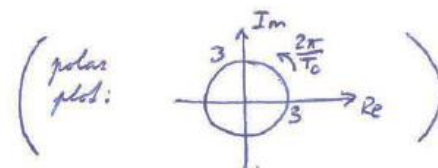
b) $x[n] = -4 \forall n$ (stem plot: )

Sol.:
$$E = \sum_{n=-\infty}^{+\infty} |x[n]|^2 = \infty \cdot 16 = \infty$$

$$P = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N |x[n]|^2 = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N 16 = \lim_{N \rightarrow \infty} \frac{2N+1}{2N+1} \cdot 16 = 16$$

$$X_{RMS} = \sqrt{P} = 4$$

c) $x(t) = 3 \cdot e^{j \frac{2\pi}{T_0} t}$



Sol.:
$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = 9 \cdot \infty = \infty$$

$$P = \frac{1}{T_0} \int_0^{T_0} |x(t)|^2 dt = \frac{1}{T_0} \int_0^{T_0} |3 \cdot e^{j \frac{2\pi}{T_0} t}|^2 dt = \frac{1}{T_0} \cdot 9 \cdot [t]_0^{T_0} = 9$$

$$X_{RMS} = \sqrt{P} = 3$$

Energy vs. power - questions and answers

Q: Is some energy signal also power signal?

A: No. If $0 < E < \infty$, then $P = 0$. If $0 < P < \infty$, then $E = \infty$.

Q: Are all signals either energy or power signals?

A: No. Any infinite duration increasing amplitude signal will not be either.
 (example: $x(t) = t^2$ is neither power signal ($P = \infty$) nor energy signal ($E = \infty$).)

Vocabulary: instantaneous value - okamžitá hodnota

time instant - časový okamžik

average value - průměrná hodnota

fundamental period - základní perioda

arbitrary - libovolný

transient phenomenon - přechodný jev

References

- Vejražka, František. Signály a soustavy / 4.vyd. Praha: ČVUT, 1996. 243 s. ISBN 80-01-00450-3., In Czech

